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# Arithmetic Progression (AP)

## 1. INTRODUCTION TO SEQUENCES

**Sequence:** An arrangement of numbers in a definite order according to some rule.

**Examples:**

- 1, 3, 5, 7, 9, ... (odd numbers)
- 2, 4, 6, 8, 10, ... (even numbers)
- 1, 4, 9, 16, 25, ... (perfect squares)

**Terms of a sequence:** Individual numbers in a sequence are called terms, denoted by $a_1$, $a_2$, $a_3$, ..., $a_n$

## 2. ARITHMETIC PROGRESSION (AP) - DEFINITION

Arithmetic Progression is a sequence of numbers in which the difference between any two consecutive terms is constant.

This constant difference is called the common difference, denoted by d.

**Mathematical Form:** a, a+d, a+2d, a+3d, ...

**Where:**

- a = first term
- d = common difference

## 3. COMMON DIFFERENCE (d)

**Formula:** $d = a_2 - a_1 = a_3 - a_2 = a_n - a_{n-1}$

**Finding Common Difference:** Subtract any term from its next term.

**Examples:**

1. AP: 3, 7, 11, 15, 19, ...
   - d = 7 - 3 = 4
   - d = 11 - 7 = 4 ✓
2. AP: 21, 18, 15, 12, 9, ...
   - d = 18 - 21 = -3
   - d = 15 - 18 = -3 ✓
3. AP: 5, 5, 5, 5, 5, ...

- $d = 5 - 5 = 0$

**Types based on d:**

- If $d > 0$ → Increasing AP
- If $d < 0$ → Decreasing AP
- If $d = 0$ → Constant AP

# 4. GENERAL TERM (nth TERM) OF AN AP

**Formula:** $a_n = a + (n-1)d$

**Where:**

- $a_n$ = nth term
- $a$ = first term
- $n$ = number of terms
- $d$ = common difference

**Derivation:**

- 1st term = $a = a + (1-1)d$
- 2nd term = $a + d = a + (2-1)d$
- 3rd term = $a + 2d = a + (3-1)d$
- nth term = $a + (n-1)d$

**Important Points:**

- To find any term, you need a, d, and n
- Last term is often denoted by 'l'

**Examples:**

**Example 1:** Find the 10th term of AP: 2, 5, 8, 11, ...

- $a = 2$, $d = 5-2 = 3$, $n = 10$
- $a_{10} = 2 + (10-1)\times 3 = 2 + 27 = 29$

**Example 2: Find the 20th term of AP: 100, 95, 90, 85, ...**

- $a = 100$, $d = 95-100 = -5$, $n = 20$
- $a_{20} = 100 + (20-1)\times(-5) = 100 - 95 = 5$

# 5. FINDING NUMBER OF TERMS IN AN AP

When last term (l) is given, **use:** $l = a + (n-1)d$

**Rearranging:** $n = [(l - a)/d] + 1$

**Example: How many terms are there in AP: 7, 10, 13, ..., 91?**

- $a = 7, d = 3, l = 91$
- $91 = 7 + (n-1)\times 3$
- $84 = (n-1)\times 3$
- $n-1 = 28$
- $n = 29$

## 6. SELECTION OF TERMS IN AN AP

For convenience in problems:

| Number of Terms | Terms to Select | Common Difference |
|---|---|---|
| 3 terms | a-d, a, a+d | d |
| 4 terms | a-3d, a-d, a+d, a+3d | 2d |
| 5 terms | a-2d, a-d, a, a+d, a+2d | d |

This makes calculations easier as the sum simplifies.

## 7. SUM OF n TERMS OF AN AP

Formula 1 (when first term and common difference are known):

$S_n = n/2 \times [2a + (n-1)d]$

Formula 2 (when first and last terms are known):

$S_n = n/2 \times [a + l]$

Where $l$ = last term = $a_n$

Derivation of Formula: Write the sum in two ways:

- $S_n = a + (a+d) + (a+2d) + ... + l$
- $S_n = l + (l-d) + (l-2d) + ... + a$

Adding both: $2S_n = (a+l) + (a+l) + ...$ n times $2S_n = n(a+l)$ $S_n = n/2 \times (a+l)$

Since $l = a + (n-1)d$: $S_n = n/2 \times [2a + (n-1)d]$

Examples:

Example 1: Find sum of first 20 terms of AP: 3, 7, 11, 15, ...

- $a = 3, d = 4, n = 20$
- $S_{20} = 20/2 \times [2\times 3 + (20-1)\times 4]$

- $S_{20} = 10 \times [6 + 76]$
- $S_{20} = 10 \times 82 = 820$

Example 2: Find sum of AP: 2, 5, 8, ..., 59

- $a = 2, d = 3, l = 59$
- First find n: $59 = 2 + (n-1)\times 3 \rightarrow n = 20$
- $S_{20} = 20/2 \times (2 + 59) = 10 \times 61 = 610$

# 8. IMPORTANT PROPERTIES OF AP

**Property 1:** If a constant is added or subtracted from each term of an AP, the resulting sequence is also an AP with the same common difference.

**Property 2:** If each term of an AP is multiplied or divided by a non-zero constant, the resulting sequence is also an AP.

**Property 3:** Three numbers a, b, c are in AP if and only if $2b = a + c$ (b is the arithmetic mean of a and c)

**Property 4:** In a finite AP, the sum of terms equidistant from the beginning and end is constant and equals (first term + last term).

- $a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} = \dots$

**Property 5:** If $a_n$ is the nth term of an AP, then:

- $S_n - S_{n-1} = a_n$ (nth term can be found from sum formula)

**Property 6:** Sum of first n natural numbers: $1 + 2 + 3 + \dots + n = n(n+1)/2$

**Property 7:** Sum of first n odd numbers: $1 + 3 + 5 + \dots + (2n-1) = n^2$

**Property 8:** Sum of first n even numbers: $2 + 4 + 6 + \dots + 2n = n(n+1)$

# 9. ARITHMETIC MEAN

Arithmetic Mean (AM) between two numbers a and b:

$AM = (a + b)/2$

Arithmetic Mean between multiple numbers: If n numbers are in AP between a and b, then:

- Common difference $d = (b - a)/(n + 1)$

Example: Insert 3 arithmetic means between 5 and 21.

- We have: 5, $A_1$, $A_2$, $A_3$, 21 (5 terms total)
- $d = (21 - 5)/(3 + 1) = 16/4 = 4$
- $A_1 = 5 + 4 = 9$

- $A_2 = 9 + 4 = 13$
- $A_3 = 13 + 4 = 17$
- AP: 5, 9, 13, 17, 21

## 10. IMPORTANT FORMULAS AT A GLANCE

| S.No. | Formula | Description |
|---|---|---|
| 1 | $d = a_n - a_{n-1}$ | Common difference |
| 2 | $a_n = a + (n-1)d$ | nth term |
| 3 | $S_n = n/2[2a + (n-1)d]$ | Sum of n terms |
| 4 | $S_n = n/2(a + l)$ | Sum with last term |
| 5 | $AM = (a+b)/2$ | Arithmetic mean |
| 6 | $2b = a + c$ | Three terms in AP |

## 11. PROBLEM-SOLVING STRATEGIES

**Step 1: Identify what is given ($a$, $d$, $n$, $a_n$, $S_n$)**

**Step 2: Identify what needs to be found**

**Step 3: Choose the appropriate formula**

**Step 4: Substitute values and solve**

**Common Problem Types:**

**Type 1: Finding nth term**

- Use: $a_n = a + (n-1)d$

**Type 2: Finding sum of n terms**

- Use: $S_n = n/2[2a + (n-1)d]$ or $S_n = n/2(a + l)$

**Type 3: Finding number of terms**

- Use: $n = [(l-a)/d] + 1$

Type 4: Checking if sequence is AP

- Check if d is constant

Type 5: Word problems

- Convert to AP form first
- Identify a, d, n
- Apply formulas

## 12. WORKED EXAMPLES

Example 1: Which term of AP 21, 18, 15, ... is -81?

- $a = 21, d = 18 - 21 = -3$
- $a_n = -81$
- $-81 = 21 + (n-1)(-3)$
- $-81 = 21 - 3n + 3$
- $-81 = 24 - 3n$
- $-105 = -3n$
- $n = 35$
- Answer: 35th term

Example 2: Find the sum of first 15 multiples of 8.

- AP: 8, 16, 24, ..., 120
- $a = 8, d = 8, n = 15$
- $S_{15} = 15/2[2×8 + (15-1)×8]$
- $S_{15} = 15/2[16 + 112]$
- $S_{15} = 15/2 × 128$
- $S_{15} = 15 × 64 = 960$
- Answer: 960

Example 3: The sum of 4th and 8th terms of an AP is 24 and the sum of 6th and 10th terms is 44. Find the first three terms.

- $a_4 + a_8 = 24$
- $(a + 3d) + (a + 7d) = 24$
- $2a + 10d = 24$
- $a + 5d = 12$ ... (i)
- $a_6 + a_{10} = 44$
- $(a + 5d) + (a + 9d) = 44$
- $2a + 14d = 44$
- $a + 7d = 22$ ... (ii)

Subtracting (i) from (ii):

- $2d = 10$
- $d = 5$

From (i): a + 5(5) = 12 → a = -13

First three terms: -13, -8, -3

Example 4: How many two-digit numbers are divisible by 3?

- First two-digit number divisible by 3 = 12
- Last two-digit number divisible by 3 = 99
- AP: 12, 15, 18, ..., 99
- a = 12, d = 3, l = 99
- 99 = 12 + (n-1)×3
- 87 = (n-1)×3
- n-1 = 29
- n = 30
- Answer: 30 numbers

Example 5: Find the sum of all three-digit natural numbers which are divisible by 7.

- First three-digit number divisible by 7 = 105
- Last three-digit number divisible by 7 = 994
- a = 105, d = 7, l = 994
- Find n: 994 = 105 + (n-1)×7
- 889 = (n-1)×7
- n = 128
- $S_n$ = n/2(a + l) = 128/2(105 + 994)
- $S_{128}$ = 64 × 1099 = 70,336
- Answer: 70,336

## 13. COMMON MISTAKES TO AVOID

1. Forgetting (n-1) in the formula for nth term
2. Not checking if sequence is actually an AP before applying formulas
3. Sign errors with negative common difference
4. Confusing number of terms with the term value
5. Not simplifying before substitution in complex problems
6. Wrong formula selection - check what is given and what is required

## 14. QUICK REVISION POINTS

✓ AP has constant difference between consecutive terms

✓ d can be positive, negative, or zero

✓ nth term: $a_n = a + (n-1)d$

✓ Sum of n terms: $S_n = n/2[2a + (n-1)d]$

✓ Alternative sum formula: $S_n = n/2(a + l)$

✓ AM of a and b = (a+b)/2

✓ Three numbers a, b, c in AP ⇒ 2b = a + c

✓ Sum of first n natural numbers = n(n+1)/2

✓ Always verify if sequence is AP before using formulas

**ALL THE BEST! 📚**

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