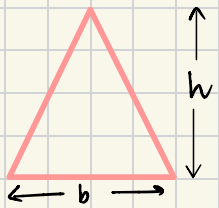
Heron's Formula

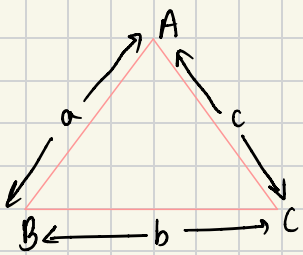


area of $\triangle = \frac{1}{2} \times b \times h$

But what if height of $\triangle$ is not given, and all sides of $\triangle$ are given!

↓

In that case we use heron's formula to find area of $\triangle$



Sides of $\triangle$ = a, b, c

Perimeter (P) = a + b + c

Semi - perimeter = $\frac{a+b+c}{2}$

$$S = \frac{a+b+c}{2}$$

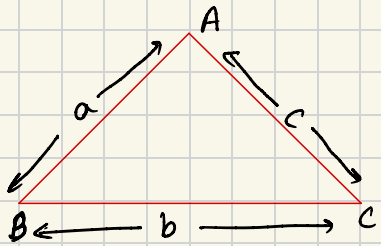
Area of $\triangle = \sqrt{S(S-a)(S-b)(S-c)}$

S = semi-perimeter

a, b, c → sides of $\triangle$

Made with Goodnotes

Q) Find area of a △, two sides of which are 8cm and 11cm & perimeter is 32cm



Let $a = 8\,cm$

$b = 11\,cm$

perimeter $= a+b+c = 32$

$\rightarrow 8+11+c = 32$

$\rightarrow c = 32-19$

$\boxed{c = 13\,cm}$

$s = \frac{p}{2} = \frac{32}{2} = \boxed{16\,cm}$

area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{16(16-8)(16-11)(16-13)}$

$= \sqrt{16 \times 8 \times 5 \times 3}$

$= \sqrt{4\times4\times4\times2\times5\times3}$

$= 4\sqrt{\frac{2\times2}{4}\times2\times5\times3}$

$= 4\times2\sqrt{2\times5\times3}$

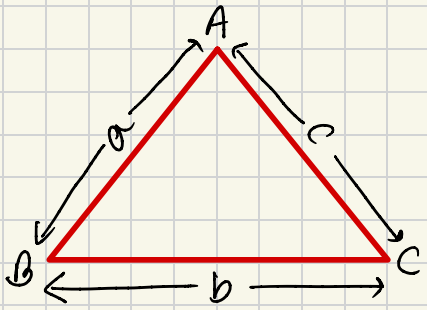
$\boxed{= 8\sqrt{30}\ cm^2}$ ✓ ans.

Made with Goodnotes

Q) The lengths of all sides of a △ are in the ratio 3:4:5 & it's perimeter is 144 cm. Find :- 1) Area of △, 2) Height corresponding to longest side

Let the sides in the ratio be $3x, 4x, 5x$

$\therefore$ $a = 3x$, $b = 4x$, $c = 5x$



$P = 144$ cm

$a + b + c = 144$

$3x + 4x + 5x = 144$

$x = \frac{144}{12}$

$x = 12$ cm

$a = 3x = 3 \times 12 = 36$ cm

$b = 4x = 4 \times 12 = 48$ cm

$c = 5x = 5 \times 12 = 60$ cm

Semi-perimeter $(s) = \frac{P}{2} = \frac{144}{2} \Rightarrow 72$ cm

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{72 \times 36 \times 24 \times 12}$

$= \sqrt{6 \times 6 \times 2 \times 6 \times 6 \times 2 \times 3 \times 2 \times 2 \times 4 \times 3}$

$= 6 \times 6 \times 2 \times 3 \times 4 = 72 \times 12$

i) Area of △ = 864

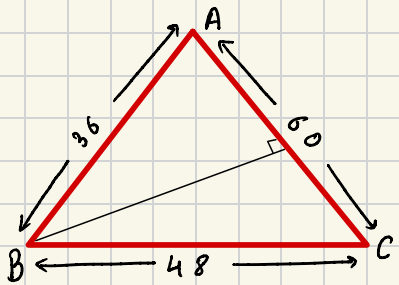
Made with Goodnotes

Area of $\triangle = \frac{1}{2} \times b \times h$

$864 = \frac{1}{2} \times 60 \times h$

$h = \frac{864}{30}$

ii) $h = 28.2$

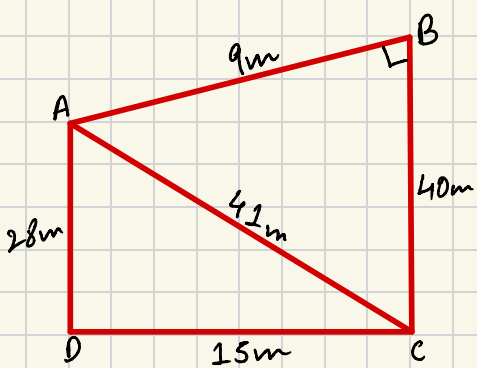


Q) Find the area ABCD in which AB = 9m, BC = 40m, ∠ABC = 90°, CD = 15m & AD = 28m

Area of $\triangle ABC = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 9 \times 40$

$= 180\ m$



In $\triangle ABC$, By pythagoreas theorem

$(AC)^2 = (AB)^2 + (BC)^2$

$= 81 + 1600$

$AC = \sqrt{1681}$

$AC = 41\ m$

Now in $\triangle ABC$, $s = \frac{28+41+15}{2} = 42m$

Area of $\triangle ACD = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{42(42-28)(42-15)(42-41)}$

Made with Goodnotes

$= \sqrt{42 \times 14 \times 27 \times 1}$

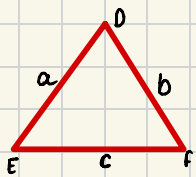
$= \sqrt{7 \times 6 \times 7 \times 2 \times 3 \times 3 \times 3}$

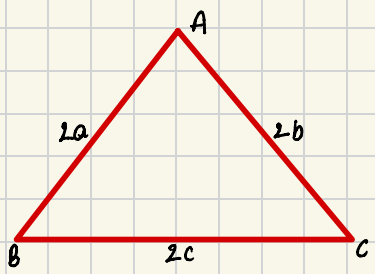
$= \sqrt{7 \times 7 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3}$

$= 7 \times 3 \times 3 \times 2$

$= 126\ m^2$ ans

Q) If each side of a △ is doubled then find the ratio of the area of new △ thus formed and the given △.





To find : $\frac{\text{Area } \triangle ABC}{\text{Area } \triangle DEF}$

For △ DEF, $S_1 = \frac{a+b+c}{2}$ —— ①

Area of △ DEF $= \sqrt{S_1(S_1-a)(S_1-b)(S_1-c)}$ —— ③

For △ ABC, $S_2 = \frac{2a+2b+2c}{2} = a+b+c$ —— ②

$S_2 = 2S_1$

Made with Goodnotes

Area of $\triangle ABC$, $\rightarrow \sqrt{S_2(S_2-2a)(S_2-2b)(S_2-2c)}$

$\rightarrow \sqrt{2S_1(2S_1-2a)(2S_1-2b)(2S_1-2c)}$

$\rightarrow \sqrt{2S_1 \times 2(S_1-a) \times 2(S_1-b) \times 2(S_1-c)}$

$\rightarrow 2\times 2\sqrt{S_1(S_1-a)(S_1-b)(S_1-c)}$

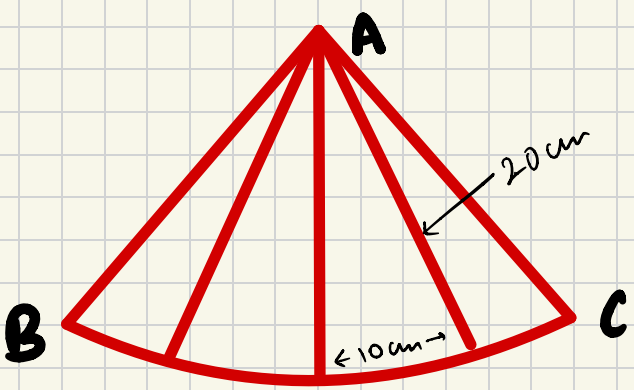
$\rightarrow 4\sqrt{S_1(S_1-a)(S_1-b)(S_1-c)}$ — (4)

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEF} = \frac{4\sqrt{S_1(S_1-a)(S_1-b)(S_1-c)}}{\sqrt{S_1(S_1-a)(S_1-b)(S_1-c)}}$$

$$= \frac{4}{1}$$

Hence, $\frac{4}{1}$ is the required ratio

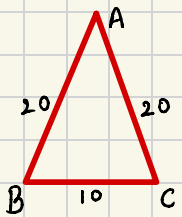
Q) A conical tent is made by stitching 12 triangular pieces of cloth of two different colours red and white alternatively, each piece measuring 10 cm, 20 cm and 20 cm, then how much cloth of red colour is required to make the conical tent ?



Made with Goodnotes

Toted no. of rectangular pieces = 12 (red + white)

Triangular pieces of red colour = 6



Dimensions of each triangle :

$a = 20$

$b = 10$

$c = 20$

$s = \frac{20+10+20}{2} = 25$

Area of $1\triangle = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{25(25-20)(25-10)(25-20)}$

$= \sqrt{25 \times 5 \times 15 \times 5}$

$= 25\sqrt{15}$

Amount of red colour cloth required to make a conical tent = 6 x area of 1 $\triangle$

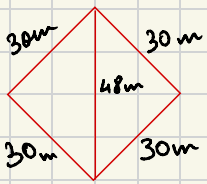
$= 6 \times 25\sqrt{15}$

$= 150\sqrt{15}$ cm2

Made with Goodnotes

Q) A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30m & its longer diagonal is 48m, how much area of grass field will each cow be getting?

Total no. of cows = 18



$s = \frac{30+30+48}{2} = \frac{108}{2}$

$s = 54$ m

Area of each triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{54(54-30)(54-30)(54-48)}$

$= \sqrt{54 \times 24 \times 24 \times 14}$

$= \sqrt{3 \times 3 \times 3 \times 2 \times 24 \times 24 \times 7 \times 2}$

$= 3 \times 24 \times 2\sqrt{21}$

$= 144\sqrt{21}$ m²

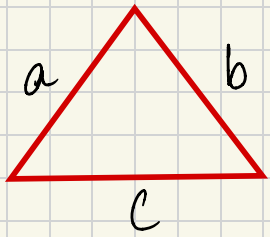
Total area of grass field = 2 × area of each △ = $2 \times 144 \times \sqrt{21}$ m²
$= 288\sqrt{21}$ m²

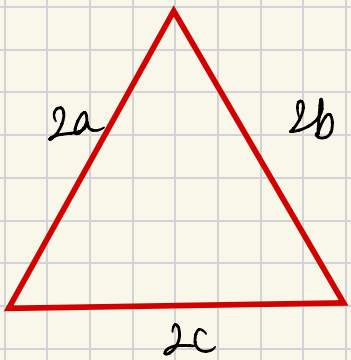
18 cows will get $288\sqrt{21}$ m² of grass field.

Each cow will get $\frac{288}{18}\sqrt{21}$ m² = $16\sqrt{21}$ m² of grass field.

Made with Goodnotes

Q) Find the percentage increase in the area of the △, if it's each side is doubled.





$$S_1 = \frac{a+b+c}{2}$$

$$S_2 = \frac{2a+2b+2c}{2} = \frac{2(a+b+c)}{2} = \frac{a+b+c}{2}$$

$$\boxed{S_2 = 2S}$$

$$A_1 = \sqrt{S_1(S_1-a)(S_1-b)(S_1-c)}$$

$$A_2 = \sqrt{S_2(S_2-2a)(S_2-2b)(S_2-2c)}$$

$$= \sqrt{2S_1(2S_1-2a)(2S_1-2b)(2S_1-2c)}$$

$$= \sqrt{2S_1 \times 2(S_1-a) \times 2(S_1-b) \times 2(S_1-c)}$$

$$= 4\sqrt{S_1(S_1-a)(S_1-b)(S_1-c)}$$

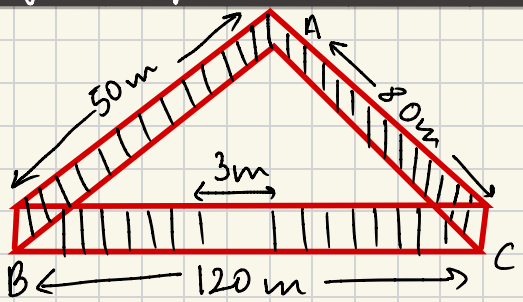
$$\boxed{A_2 = 4A_1}$$

$$\text{Increase} = \frac{\text{New area} - \text{original area}}{\text{Original area}} \times 100 = \frac{A_2 - A_1}{A_1} \times 100 = \frac{4A_1 - A_1}{A_1} \times 100$$

$$= 300\%$$

Made with Goodnotes

Q) A triangular park ABC has sides 120m, 80m, 50m. A gardener has to put a fence all around it and also plant grass inside. How much area does she need to plant? Find the cost of fencing it with barbed wire at the rate of ₹20/m leaving a space 3m wide for a gate on one side.



$a = 120\ m$
$b = 50\ m$
$c = 80\ m$

$s = \frac{120 + 50 + 80}{2}$

$s = 125\ m$

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{125(125-120)(125-50)(125-80)}\ m^2$

$= \sqrt{125 \times 5 \times 75 \times 45} \Rightarrow \sqrt{5 \times 25 \times 5 \times 3 \times 25 \times 9 \times 5}$

$= 25 \times 5 \times 3\sqrt{15}$

$= 375\sqrt{15}\ m^2$

$\therefore 375\sqrt{15}\ m^2$ area she need to plant.

Total length to be fenced $= 120 + 50 + 80 - 3 = 147\ m$

Cost to fence 147 m $= ₹\ 20 \times 147 = ₹\ 2940$

Made with Goodnotes

Q) The triangular wall of a flyover has been used for advertisements. The sides of the wall are 13m, 14m & 15m. The advertisements yield an earning of ₹2000/m² an year. A company hired one of its walls for 6 months. How much rent did it pay?

The sides of the walls are 13m, 14 m & 15 m.

The advertisements yield an earning of ₹2000/m² an year

We have to calculate the rent paid by the company for 6 months.

Let $a = 13\,m$
$b = 14\,m$
$c = 15\,m$

By heron's formula

$$s = \frac{a+b+c}{2} = \frac{13+14+15}{2} = 21\,m$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{21\,(21-13)(21-14)(21-15)}$$

$$= \sqrt{21 \times 8 \times 7 \times 6} = \sqrt{7 \times 3 \times 4 \times 2 \times 7 \times 2 \times 3}$$

$$= 7 \times 4 \times 3 = 84\,m^2$$

Area = $84\,m^2$ ✓

Cost of advertising per year for $1\,m^2$ = ₹2000

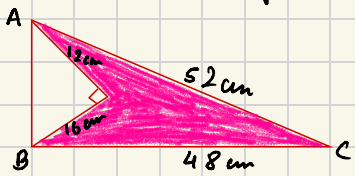
Advertisement cost / year for $84\,m^2$ = ₹2000 × 84 = ₹168000

Rent paid by company : ₹168000/2 = ₹84000/- ✓

Homework Questions

Q1) The perimeter of an isosceles triangle is 42 cm and its base is (3/2) times each of the equal sides. Find the length of each side of the △, area and height of the △.

Q2) Find the area of the shaded region in fig.

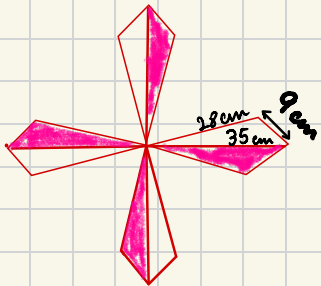


Q3) In a △ ABC, AB = 15cm, BC = 13 cm, AC = 14cm. Find the area of △ABC and its altitude on AC

Q4) The perimeter of a △ field is 540m and its sides are in the ratio 25:17:12. Find the area of △.

Q5) Find the area of a △ two sides of which are 18cm & 10 cm, perimeter is 42 cm.

Q6) A floral design on a floor is made up of 6 tiles which are △, the sides of the triangle being 9cm, 28cm & 35 cm, find the cost of polishing the tiles at the rate of 50p/cm².



Made with Goodnotes

Made with Goodnotes