StudyByU

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

1. LINEAR EQUATION IN TWO VARIABLES

1.1 Definition

Linear Equation in Two Variables: An equation that can be written in the form:

$ax + by + c = 0$

Where:

- **a, b, c are real numbers**
- **$a \neq 0$ and $b \neq 0$ (at least one of a or b must be non-zero)**
- **x and y are variables**

1.2 Standard Form

General Form: $ax + by + c = 0$

Examples:

$2x + 3y - 5 = 0$ ✓ (Linear equation)

$x - y + 7 = 0$ ✓ (Linear equation)

$5x + 2y = 10$ ✓ (Can be written as $5x + 2y - 10 = 0$)

Not Linear Equations:

$x^2 + y = 5$ ✗ (x has power 2)

$xy + 2 = 0$ ✗ (x and y are multiplied)

$1/x + y = 3$ ✗ (x is in denominator)

$\sqrt{x} + y = 4$ ✗ (x is under square root)

1.3 Solution of Linear Equation

Solution: A pair of values (x, y) that satisfies the equation (makes it true).

Example: For equation: $2x + 3y = 12$

Solutions:

- **(0, 4) is a solution because: $2(0) + 3(4) = 0 + 12 = 12$ ✓**
- **(3, 2) is a solution because: $2(3) + 3(2) = 6 + 6 = 12$ ✓**
- **(6, 0) is a solution because: $2(6) + 3(0) = 12 + 0 = 12$ ✓**
- **(1, 3) is NOT a solution because: $2(1) + 3(3) = 2 + 9 = 11 \neq 12$ ✗**

IMPORTANT: A single linear equation in two variables has INFINITELY MANY SOLUTIONS.

1.4 Graph of Linear Equation

Key Point: The graph of a linear equation in two variables is always a STRAIGHT LINE.

How to draw:

1. **Find at least 2 solutions (better to find 3 for accuracy)**
2. **Plot these points on graph paper**
3. **Join them with a straight line**
4. **Extend the line on both sides**

Example: Draw graph of $x + y = 5$

Solution:

x	y	Point (x, y)
0	5	(0, 5)
5	0	(5, 0)
2	3	(2, 3)

Plot these points and join them to get a straight line.

2. PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

2.1 Definition

Pair of Linear Equations: Two linear equations considered together.

General Form:

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where:

- $a_1, b_1, c_1, a_2, b_2, c_2$ are real numbers
- $a_1^2 + b_1^2 \neq 0$ and $a_2^2 + b_2^2 \neq 0$

Example:

$2x + 3y = 7$

$3x - y = 5$

2.2 Solution of Pair of Linear Equations

Solution of a Pair: Values of x and y that satisfy BOTH equations simultaneously.

Example: For the pair:

$x + y = 5$

$x - y = 1$

Solution: $x = 3, y = 2$

- Check Equation 1: $3 + 2 = 5$ ✓
- Check Equation 2: $3 - 2 = 1$ ✓

Both equations are satisfied, so (3, 2) is the solution.

3. GRAPHICAL METHOD OF SOLUTION

3.1 Principle

Since each linear equation represents a straight line, a pair of linear equations is represented by TWO LINES.

The solution depends on how these lines behave:

1. Lines intersect at one point → ONE unique solution (Consistent)
2. Lines are parallel → NO solution (Inconsistent)
3. Lines coincide (same line) → INFINITELY MANY solutions (Dependent/Consistent)

3.2 Steps for Graphical Solution

Step 1: Express each equation in the form $y = mx + c$ (if needed)

Step 2: Find at least 3 points for each equation

Step 3: Plot the points on graph paper for both equations

Step 4: Draw the lines by joining the points

Step 5: Observe the lines:

- **If they intersect → Point of intersection is the solution**
- **If they are parallel → No solution**
- **If they coincide → Infinitely many solutions**

3.3 Example

Solve graphically:

x + y = 5

x - y = 1

Solution:

For x + y = 5:

x	0	2	5
y	5	3	0

For x - y = 1:

x	0	2	4
y	-1	1	3

Plot both lines. They intersect at point (3, 2).

Solution: x = 3, y = 2

4. ALGEBRAIC CONDITIONS FOR CONSISTENCY

4.1 Comparing Ratios

For the pair of equations:

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Compare the ratios: a_1/a_2, b_1/b_2, c_1/c_2

4.2 Three Cases

Condition	Graphical Representation	Solution	Name
$a_1/a_2 \neq b_1/b_2$	Lines intersect at one point	One unique solution (Consistent)	Consistent Pair
$a_1/a_2 = b_1/b_2 \neq c_1/c_2$	Lines are parallel	No solution (Inconsistent)	Inconsistent Pair
$a_1/a_2 = b_1/b_2 = c_1/c_2$	Lines coincide (same line)	Infinitely many solutions (Dependent)	Dependent Pair

4.3 Examples

Example 1: Check consistency

$2x + 3y = 7$ ... (1)

$4x + 6y = 14$ ... (2)

Solution:

- $a_1 = 2$, $b_1 = 3$, $c_1 = -7$
- $a_2 = 4$, $b_2 = 6$, $c_2 = -14$

Compare ratios:

- $a_1/a_2 = 2/4 = 1/2$
- $b_1/b_2 = 3/6 = 1/2$
- $c_1/c_2 = -7/-14 = 1/2$

Since $a_1/a_2 = b_1/b_2 = c_1/c_2$, the equations have INFINITELY MANY SOLUTIONS (Dependent pair).

Example 2: **Check consistency**

$2x + 3y = 7$ ... (1)

$4x + 6y = 10$ ... (2)

Solution:

- $a_1/a_2 = 2/4 = 1/2$
- $b_1/b_2 = 3/6 = 1/2$
- $c_1/c_2 = -7/-10 = 7/10$

Since $a_1/a_2 = b_1/b_2 \neq c_1/c_2$, the equations have NO SOLUTION (Inconsistent pair).

Example 3: **Check consistency**

$2x + 3y = 7$ ... (1)

$3x + 4y = 10$ ... (2)

Solution:

- $a_1/a_2 = 2/3$
- $b_1/b_2 = 3/4$

Since $a_1/a_2 \neq b_1/b_2$, the equations have ONE UNIQUE SOLUTION (Consistent pair).

4.4 Summary Table

Ratio Condition	Lines	Solutions	Type
$a_1/a_2 \neq b_1/b_2$	Intersecting	Unique solution	Consistent
$a_1/a_2 = b_1/b_2 \neq c_1/c_2$	Parallel	No solution	Inconsistent
$a_1/a_2 = b_1/b_2 = c_1/c_2$	Coincident	Infinite solutions	Dependent/Consistent

5. ALGEBRAIC METHODS OF SOLUTION

There are TWO main algebraic methods (as per CBSE 2025-26):

1. **Substitution Method**
2. **Elimination Method**

Note: Cross-multiplication method is DELETED from syllabus 2025-26.

6. SUBSTITUTION METHOD

6.1 Principle

Express one variable in terms of the other from one equation, then substitute this value in the second equation.

6.2 Steps

Step 1: Choose one equation (preferably simpler one)

Step 2: Express one variable in terms of the other

- **Either x in terms of y: $x = \text{(expression in y)}$**
- **Or y in terms of x: $y = \text{(expression in x)}$**

Step 3: Substitute this expression in the second equation

Step 4: Solve for the remaining variable

Step 5: Substitute back to find the other variable

Step 6: Verify by putting values in both original equations

6.3 Example 1 (Simple)

Solve:

$x + y = 7$ ... (1)

$x - y = 1$ ... (2)

Solution:

From equation (1): $x = 7 - y$ ... (3)

Substitute (3) in equation (2):

$(7 - y) - y = 1$

$7 - 2y = 1$

$-2y = 1 - 7$

$-2y = -6$

$y = 3$

Substitute y = 3 in equation (3):

x = 7 - 3 = 4

Answer: x = 4, y = 3

Verification:

- **Equation (1): 4 + 3 = 7 ✓**
- **Equation (2): 4 - 3 = 1 ✓**

6.4 Example 2 (With Fractions)

Solve:

2x + 3y = 11 ... (1)

2x - 4y = -24 ... (2)

Solution:

From equation (1):

2x = 11 - 3y

x = (11 - 3y)/2 ... (3)

Substitute (3) in equation (2):

2[(11 - 3y)/2] - 4y = -24

11 - 3y - 4y = -24

11 - 7y = -24

-7y = -24 - 11

-7y = -35

y = 5

Substitute y = 5 in equation (3):

x = (11 - 3(5))/2

x = (11 - 15)/2

x = -4/2

x = -2

Answer: $x = -2$, $y = 5$

6.5 When to Use Substitution Method

Best used when:

- One variable has coefficient 1 or -1
- One equation is already solved for a variable
- Equations are simple

7. ELIMINATION METHOD

7.1 Principle

Eliminate one variable by making coefficients of that variable equal (by multiplication if needed), then add or subtract equations.

7.2 Steps

Step 1: Choose a variable to eliminate (x or y)

Step 2: Make coefficients of that variable equal in both equations

- Multiply equations by suitable numbers if needed

Step 3: Add or subtract equations to eliminate the variable

- If coefficients have same sign → Subtract
- If coefficients have opposite signs → Add

Step 4: Solve for the remaining variable

Step 5: Substitute back in any original equation to find the other variable

Step 6: Verify the solution

7.3 Example 1 (Direct Elimination)

Solve:

$3x + 2y = 11$ ... (1)

$3x - 2y = 1$ ... (2)

Solution:

Notice: Coefficients of x are already equal (3 and 3)

Add equations (1) and (2):

(3x + 2y) + (3x - 2y) = 11 + 1

6x = 12

x = 2

Substitute x = 2 in equation (1):

3(2) + 2y = 11

6 + 2y = 11

2y = 5

y = 5/2 = 2.5

Answer: x = 2, y = 2.5

7.4 Example 2 (With Multiplication)

Solve:

2x + 3y = 11 ... (1)

3x + 2y = 9 ... (2)

Solution:

To eliminate x, make coefficients equal:

- LCM of 2 and 3 = 6
- Multiply (1) by 3: 6x + 9y = 33 ... (3)
- Multiply (2) by 2: 6x + 4y = 18 ... (4)

Subtract (4) from (3):

(6x + 9y) - (6x + 4y) = 33 - 18

5y = 15

y = 3

Substitute y = 3 in equation (1):

2x + 3(3) = 11

2x + 9 = 11

2x = 2

x = 1

Answer: x = 1, y = 3

7.5 Example 3 (Eliminating y)

Solve:

5x - 4y = 3 ... (1)

3x - 2y = 5 ... (2)

Solution:

To eliminate y, make coefficients equal:

- LCM of 4 and 2 = 4
- Keep (1) as is: 5x - 4y = 3
- Multiply (2) by 2: 6x - 4y = 10 ... (3)

Subtract (1) from (3):

(6x - 4y) - (5x - 4y) = 10 - 3

x = 7

Substitute x = 7 in equation (2):

3(7) - 2y = 5

21 - 2y = 5

-2y = -16

y = 8

Answer: x = 7, y = 8

7.6 When to Use Elimination Method

Best used when:

- Coefficients can be easily made equal
- No variable has coefficient 1
- Working with whole numbers

8. COMPARISON: SUBSTITUTION vs ELIMINATION

Aspect	Substitution Method	Elimination Method
Best for	When coefficient of x or y is 1	When coefficients can be made equal easily
Process	Express one variable, substitute	Make coefficients equal, add/subtract
Calculation	May involve fractions early	Usually avoids fractions initially
Steps	5-6 steps	4-5 steps
Preferred when	One equation is simple	Both equations have similar structure

9. EQUATIONS REDUCIBLE TO LINEAR EQUATIONS

9.1 Concept

Some equations that are NOT linear can be reduced to linear form by substitution.

Common Forms:

$a/(x) + b/(y) = c$

$a/(x-p) + b/(y-q) = c$

$ax/(x+y) + by/(x-y) = c$

9.2 Method

Step 1: Substitute: Let $1/x = u$ and $1/y = v$ (or appropriate substitution)

Step 2: Convert to linear equations in u and v

Step 3: Solve for u and v using substitution or elimination

Step 4: Find x and y from u and v

9.3 Example 1

Solve:

2/x + 3/y = 13

5/x - 4/y = -2

Solution:

Step 1: Let 1/x = u and 1/y = v

Equations become:

2u + 3v = 13 ... (1)

5u - 4v = -2 ... (2)

Step 2: Solve using elimination

Multiply (1) by 4: 8u + 12v = 52 ... (3) Multiply (2) by 3: 15u - 12v = -6 ... (4)

Add (3) and (4):

23u = 46

u = 2

Substitute u = 2 in (1):

2(2) + 3v = 13

4 + 3v = 13

3v = 9

v = 3

Step 3: Find x and y

u = 1/x = 2 → x = 1/2

v = 1/y = 3 → y = 1/3

Answer: x = 1/2, y = 1/3

9.4 Example 2

Solve:

10/(x+y) + 2/(x-y) = 4

15/(x+y) - 5/(x-y) = -2

Solution:

Step 1: Let 1/(x+y) = u and 1/(x-y) = v

Equations become:

10u + 2v = 4 ... (1)

15u - 5v = -2 ... (2)

Step 2: Simplify Divide (1) by 2: 5u + v = 2 ... (3)

Step 3: Solve From (3): v = 2 - 5u ... (4)

Substitute in (2):

15u - 5(2 - 5u) = -2

15u - 10 + 25u = -2

40u = 8

u = 1/5

From (4):

v = 2 - 5(1/5) = 2 - 1 = 1

Step 4: Find x and y

1/(x+y) = 1/5 → x + y = 5 ... (5)

1/(x-y) = 1 → x - y = 1 ... (6)

Add (5) and (6):

2x = 6

x = 3

From (5): y = 5 - 3 = 2

Answer: x = 3, y = 2

10. WORD PROBLEMS

10.1 General Strategy

Step 1: Read the problem carefully

Step 2: Identify what you need to find (unknowns)

Step 3: Represent unknowns as x and y

Step 4: Write TWO equations based on given conditions

Step 5: Solve using substitution or elimination

Step 6: Check if solution satisfies the problem

Step 7: Write the answer in words

10.2 Types of Word Problems

Type 1: Age Problems

Example: Five years ago, a man was three times as old as his son. Five years hence, he will be twice as old as his son. Find their present ages.

Solution:

Let present age of man = x years Let present age of son = y years

Five years ago:

- Man's age = $x - 5$
- Son's age = $y - 5$
- Condition: $x - 5 = 3(y - 5)$
- Equation 1: $x - 5 = 3y - 15$
- Simplify: $x - 3y = -10$ ... (1)

Five years hence:

- Man's age = $x + 5$
- Son's age = $y + 5$
- Condition: $x + 5 = 2(y + 5)$
- Equation 2: $x + 5 = 2y + 10$
- Simplify: $x - 2y = 5$ ... (2)

Solve: Subtract (1) from (2):

$(x - 2y) - (x - 3y) = 5 - (-10)$

$y = 15$

Substitute in (2):

$x - 2(15) = 5$

$x = 35$

Answer: Man's present age = 35 years, Son's present age = 15 years

Type 2: Number Problems

Example: The sum of two numbers is 25 and their difference is 5. Find the numbers.

Solution:

Let the two numbers be x and y.

Given:

- **$x + y = 25$ ... (1)**
- **$x - y = 5$ ... (2)**

Add (1) and (2):

$2x = 30$

$x = 15$

From (1): $y = 25 - 15 = 10$

Answer: The numbers are 15 and 10.

Type 3: Cost Problems

Example: The cost of 2 pencils and 3 erasers is ₹9. The cost of 4 pencils and 6 erasers is ₹18. Find the cost of each.

Solution:

Let cost of 1 pencil = ₹x Let cost of 1 eraser = ₹y

Given:

- **$2x + 3y = 9$ ... (1)**
- **$4x + 6y = 18$ ... (2)**

Compare ratios:

- **$a_1/a_2 = 2/4 = 1/2$**
- **$b_1/b_2 = 3/6 = 1/2$**
- **$c_1/c_2 = 9/18 = 1/2$**

Since $a_1/a_2 = b_1/b_2 = c_1/c_2$, equations are dependent.

Answer: Infinite solutions exist. We cannot find unique cost. (Both equations represent same condition)

Type 4: Speed/Distance Problems

Example: A boat goes 30 km upstream and 44 km downstream in 10 hours. It goes 40 km upstream and 55 km downstream in 13 hours. Find the speed of boat in still water and speed

of stream.

Solution:

Let speed of boat in still water = x km/h Let speed of stream = y km/h

Then:

- Speed upstream = (x - y) km/h
- Speed downstream = (x + y) km/h

First condition:

Time upstream + Time downstream = 10

$30/(x-y) + 44/(x+y) = 10$ ... (1)

Second condition:

$40/(x-y) + 55/(x+y) = 13$ ... (2)

Let $1/(x-y) = u$ and $1/(x+y) = v$

Equations become:

$30u + 44v = 10$ ... (3)

$40u + 55v = 13$ ... (4)

Multiply (3) by 4: $120u + 176v = 40$ ... (5) Multiply (4) by 3: $120u + 165v = 39$ ... (6)

Subtract (6) from (5):

$11v = 1$

$v = 1/11$

Substitute in (3):

$30u + 44(1/11) = 10$

$30u + 4 = 10$

$30u = 6$

$u = 1/5$

Now:

$1/(x-y) = 1/5 \rightarrow x - y = 5$ ... (7)

$1/(x+y) = 1/11 \rightarrow x + y = 11$ ... (8)

Add (7) and (8):

$2x = 16$

$x = 8$

From (8): $y = 11 - 8 = 3$

Answer: Speed of boat = 8 km/h, Speed of stream = 3 km/h

Type 5: Digit Problems

Example: The sum of a two-digit number and the number obtained by reversing the digits is 66. If the digits differ by 2, find the number.

Solution:

Let the two-digit number be $10x + y$ (where x = tens digit, y = units digit)

Number obtained by reversing = $10y + x$

Condition 1: Sum is 66

$(10x + y) + (10y + x) = 66$

$11x + 11y = 66$

$x + y = 6$ ... (1)

Condition 2: Digits differ by 2

Case 1: $x - y = 2$ ... (2)

Add (1) and (2):

$2x = 8$

$x = 4$

$y = 2$

Number = 42

Case 2: $y - x = 2$ ... (3)

Add (1) and (3):

$2y = 8$

y = 4

x = 2

Number = 24

Answer: The numbers are 42 or 24.

Type 6: Fraction Problems

Example: A fraction becomes 1/3 when 1 is subtracted from both numerator and denominator. It becomes 1/4 when 8 is added to both. Find the fraction.

Solution:

Let the fraction be x/y

Condition 1:

$(x-1)/(y-1) = 1/3$

$3(x-1) = y-1$

$3x - 3 = y - 1$

$3x - y = 2$ ... (1)

Condition 2:

$(x+8)/(y+8) = 1/4$

$4(x+8) = y+8$

$4x + 32 = y + 8$

$4x - y = -24$ ... (2)

Subtract (1) from (2):

$(4x - y) - (3x - y) = -24 - 2$

$x = -26$

Substitute in (1):

$3(-26) - y = 2$

$-78 - y = 2$

$y = -80$

Answer: The fraction is -26/-80 = 13/40

(Note: Usually fractions are positive, so check problem statement)

11. IMPORTANT FORMULAS AND POINTS

11.1 Key Formulas

General Form:

$ax + by + c = 0$

Condition for Consistency:

Unique solution: $a_1/a_2 \neq b_1/b_2$

No solution: $a_1/a_2 = b_1/b_2 \neq c_1/c_2$

Infinite solutions: $a_1/a_2 = b_1/b_2 = c_1/c_2$

11.2 Quick Reference

Common Relationships:

Age Problems:

- Present age = x
- n years ago = x - n
- n years hence = x + n

Speed Problems:

- Upstream speed = (boat speed - stream speed)
- Downstream speed = (boat speed + stream speed)
- Time = Distance/Speed

Number Problems:

- Two-digit number = 10x + y (x = tens, y = units)
- Reversed number = 10y + x

Fraction Problems:

- Original fraction = x/y
- After changes = $(x \pm a)/(y \pm b)$

12. STEP-BY-STEP PROBLEM SOLVING

12.1 For Algebraic Method Problems

Checklist:

1. □ **Write both equations clearly**
2. □ **Decide method (substitution or elimination)**
3. □ **Show all steps clearly**
4. □ **Solve systematically**
5. □ **Verify answer in both equations**
6. □ **Write final answer with units (if applicable)**

12.2 For Word Problems

Checklist:

1. □ **Read problem twice**
2. □ **Identify unknowns (what to find)**
3. □ **Let x and y represent unknowns**
4. □ **Form TWO equations from given conditions**
5. □ **Solve using appropriate method**
6. □ **Check if answer makes sense logically**
7. □ **Write answer in words/complete sentence**

12.3 For Graphical Method

Checklist:

1. □ **Find at least 3 points for each equation**
2. □ **Use proper scale on graph paper**
3. □ **Plot points accurately**
4. □ **Draw straight lines with ruler**
5. □ **Mark point of intersection clearly**
6. □ **Read coordinates carefully**
7. □ **Verify algebraically**