StudyByU

TRIANGLES

1. INTRODUCTION TO SIMILAR FIGURES

Similar Figures: Two figures having the same shape but not necessarily the same size are called similar figures.

Examples:

- All circles are similar
- All squares are similar
- All equilateral triangles are similar
- Photographs of different sizes of the same object

Key Point: Similar figures have the same shape but may differ in size.

2. SIMILARITY OF TRIANGLES

Definition: Two triangles are said to be similar if:

1. Their corresponding angles are equal, AND
2. Their corresponding sides are in the same ratio (proportional)

Notation: $\triangle ABC \sim \triangle PQR$ (read as: triangle ABC is similar to triangle PQR)

Conditions for Similarity:

- $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$
- $AB/PQ = BC/QR = CA/RP$

3. CRITERIA FOR SIMILARITY OF TRIANGLES

AAA (Angle-Angle-Angle) Similarity Criterion

Statement: If in two triangles, corresponding angles are equal, then the triangles are similar.

If $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$ Then $\triangle ABC \sim \triangle PQR$

Note: Since sum of angles in a triangle is 180°, if two angles are equal, the third angle is automatically equal.

AA (Angle-Angle) Similarity Criterion

Statement: If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

This is the most commonly used criterion.

If ∠A = ∠P and ∠B = ∠Q Then △ABC ~ △PQR

SSS (Side-Side-Side) Similarity Criterion

Statement: If the corresponding sides of two triangles are proportional, then they are similar.

If AB/PQ = BC/QR = CA/RP Then △ABC ~ △PQR

Example: If AB = 6 cm, BC = 8 cm, CA = 10 cm And PQ = 3 cm, QR = 4 cm, RP = 5 cm Then AB/PQ = 6/3 = 2, BC/QR = 8/4 = 2, CA/RP = 10/5 = 2 Since all ratios are equal, △ABC ~ △PQR

SAS (Side-Angle-Side) Similarity Criterion

Statement: If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, then the two triangles are similar.

If ∠A = ∠P and AB/PQ = AC/PR Then △ABC ~ △PQR

4. IMPORTANT PROPERTIES OF SIMILAR TRIANGLES

Property 1: The ratio of any two corresponding sides in two similar triangles is always the same and is called the scale factor or ratio of similarity.

Property 2: If two triangles are similar, then the ratio of their corresponding sides is equal to the ratio of their corresponding altitudes, medians, angle bisectors, and perimeters.

If △ABC ~ △PQR, then:

- AB/PQ = BC/QR = CA/RP = (altitude of △ABC)/(altitude of △PQR) = (median)/(median) = (perimeter of △ABC)/(perimeter of △PQR)

Property 3: Congruent triangles are always similar, but similar triangles need not be congruent.

Property 4: Similarity of triangles is:

- Reflexive: △ABC ~ △ABC
- Symmetric: If △ABC ~ △PQR, then △PQR ~ △ABC
- Transitive: If △ABC ~ △PQR and △PQR ~ △XYZ, then △ABC ~ △XYZ

5. AREAS OF SIMILAR TRIANGLES THEOREM

THEOREM 6.6 (MOST IMPORTANT)

Statement: The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Formula: If $\triangle ABC \sim \triangle PQR$, then:

$$\text{Area}(\triangle ABC)/\text{Area}(\triangle PQR) = (AB/PQ)^2 = (BC/QR)^2 = (CA/RP)^2$$

Proof Outline:

- Draw altitudes AM and PN from A and P respectively
- Area of $\triangle ABC = 1/2 \times BC \times AM$
- Area of $\triangle PQR = 1/2 \times QR \times PN$
- Dividing and using similarity properties leads to the result

Example: If $\triangle ABC \sim \triangle DEF$ and AB = 4 cm, DE = 6 cm, Area($\triangle ABC$) = 16 cm^2 Find Area($\triangle DEF$).

Solution:

- $\text{Area}(\triangle ABC)/\text{Area}(\triangle DEF) = (AB/DE)^2$
- $16/\text{Area}(\triangle DEF) = (4/6)^2$
- $16/\text{Area}(\triangle DEF) = 16/36$
- $\text{Area}(\triangle DEF) = 36\ cm^2$

6. BASIC PROPORTIONALITY THEOREM (THALES THEOREM)

THEOREM 6.1

Statement: If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

Given: In $\triangle ABC$, DE ∥ BC

To Prove: AD/DB = AE/EC

Converse is also true: If AD/DB = AE/EC, then DE ∥ BC

Important Applications: This theorem is the foundation for many other theorems and is extensively used in problems.

Example: In $\triangle ABC$, DE ∥ BC. If AD = 4 cm, DB = 6 cm, AE = 3 cm, find EC.

Solution:

- AD/DB = AE/EC
- 4/6 = 3/EC
- $EC = (3 \times 6)/4 = 4.5$ cm

7. CRITERIA FOR SIMILARITY IN RIGHT TRIANGLES

THEOREM 6.7

Statement: **If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, then the triangles on each side of the perpendicular are similar to the whole triangle and to each other.**

Given: △ABC where ∠B = 90° and BD ⊥ AC

To Prove:

- △ADB ~ △ABC
- △BDC ~ △ABC
- △ADB ~ △BDC

This leads to important results:

- $BD^2 = AD \times DC$
- $AB^2 = AD \times AC$
- $BC^2 = CD \times AC$

8. PYTHAGORAS THEOREM

THEOREM 6.8

Statement: **In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.**

Formula: In △ABC, if ∠B = 90°, then:

$AC^2 = AB^2 + BC^2$

Proof: Uses similarity of triangles and areas.

Example: In a right triangle, if one leg is 5 cm and hypotenuse is 13 cm, find the other leg.

Solution:

- Let other leg = x
- $13^2 = 5^2 + x^2$
- $169 = 25 + x^2$
- $x^2 = 144$
- x = 12 cm

CONVERSE OF PYTHAGORAS THEOREM

THEOREM 6.9

Statement: In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

Given: In △ABC, $AC^2 = AB^2 + BC^2$

To Prove: ∠B = 90°

Application: This theorem is used to check whether a triangle is right-angled or not.

Example: Check if a triangle with sides 8 cm, 15 cm, and 17 cm is right-angled.

Solution:

- $17^2 = 289$
- $8^2 + 15^2 = 64 + 225 = 289$
- Since $17^2 = 8^2 + 15^2$, the triangle is right-angled.

9. IMPORTANT THEOREMS SUMMARY

Theorem	Statement	Application
Basic Proportionality Theorem	Line parallel to one side divides other sides proportionally	Finding unknown sides
AAA/AA Similarity	Equal angles → Similar triangles	Proving similarity
SSS Similarity	Proportional sides → Similar triangles	Proving similarity
SAS Similarity	One equal angle + proportional sides → Similar	Proving similarity
Area Theorem	Area ratio = $(\text{side ratio})^2$	Finding areas
Pythagoras Theorem	Hypotenuse^2 = Sum of squares of other sides	Right triangles

10. IMPORTANT RESULTS AND FORMULAS

1. Ratio of Perimeters: If △ABC ~ △PQR, then: Perimeter(△ABC)/Perimeter(△PQR) = AB/PQ

2. Ratio of Areas: If △ABC ~ △PQR, then: Area(△ABC)/Area(△PQR) = $(AB/PQ)^2$

3. Ratio of Altitudes: If △ABC ~ △PQR, then: Altitude(△ABC)/Altitude(△PQR) = AB/PQ

4. Ratio of Medians: If △ABC ~ △PQR, then: Median(△ABC)/Median(△PQR) = AB/PQ

5. Scale Factor: If △ABC ~ △PQR with ratio k, then:

- Corresponding sides are in ratio k:1
- Corresponding altitudes, medians are in ratio k:1
- Perimeters are in ratio k:1
- Areas are in ratio k^2:1

11. SPECIAL TRIANGLES

Equilateral Triangle:

- All angles = 60°
- All sides equal
- Height = $(\sqrt{3}/2) \times$ side
- Area = $(\sqrt{3}/4) \times \text{side}^2$

Isosceles Right Triangle:

- Two equal sides, one right angle
- Angles: 90°, 45°, 45°
- If equal sides = a, hypotenuse = $a\sqrt{2}$

30°-60°-90° Triangle:

- Sides are in ratio $1 : \sqrt{3} : 2$
- If shortest side = a, then sides are a, $a\sqrt{3}$, 2a

12. COMMON PROBLEM TYPES

Type 1: Proving Triangles Similar

Method:

- Identify equal angles (AA criterion)
- Or show proportional sides (SSS criterion)
- Or one equal angle + proportional sides (SAS criterion)

Example: In △ABC, D and E are points on AB and AC such that DE ∥ BC. Prove that △ADE ~ △ABC.

Solution:

- ∠A = ∠A (common)
- ∠ADE = ∠ABC (corresponding angles, DE ∥ BC)
- By AA criterion, △ADE ~ △ABC

Type 2: Finding Unknown Sides

Method:

- Establish similarity
- Use proportionality of sides
- Cross multiply and solve

Example: In △ABC, DE ∥ BC, AD = 2 cm, DB = 3 cm, AE = 3 cm. Find AC.

Solution:

- By BPT: AD/DB = AE/EC
- 2/3 = 3/EC
- EC = 4.5 cm
- AC = AE + EC = 3 + 4.5 = 7.5 cm

Type 3: Finding Areas

Method:

- Establish similarity
- Use Area(△1)/Area(△2) = $(side_1/side_2)^2$

Example: △ABC ~ △PQR. If AB = 6 cm, PQ = 9 cm, Area(△ABC) = 48 cm^2. Find Area(△PQR).

Solution:

- Area(△ABC)/Area(△PQR) = $(AB/PQ)^2$
- 48/Area(△PQR) = $(6/9)^2$
- 48/Area(△PQR) = 4/9
- Area(△PQR) = 108 cm^2

Type 4: Pythagoras Theorem Problems

Method:

- Identify right angle
- Apply $a^2 + b^2 = c^2$
- Solve for unknown

Example: A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from the base of the wall.

Solution:

- Hypotenuse = 10 m, height = 8 m
- Let distance = x
- $10^2 = 8^2 + x^2$
- $100 = 64 + x^2$
- $x^2 = 36$
- x = 6 m

Type 5: Converse of Pythagoras

Method:

- Check if $a^2 + b^2 = c^2$ (c is longest side)
- If yes, triangle is right-angled
- If $a^2 + b^2 > c^2$, triangle is acute-angled
- If $a^2 + b^2 < c^2$, triangle is obtuse-angled

13. WORKED EXAMPLES

Example 1: In △ABC, D is a point on AB such that AD = 4 cm, BD = 6 cm and AC = 15 cm. If DE ∥ BC (where E is on AC), find AE.

Solution: By Basic Proportionality Theorem:

- AD/BD = AE/EC
- Also, AD/AB = AE/AC
- 4/(4+6) = AE/15
- 4/10 = AE/15
- AE = (4 × 15)/10 = 6 cm

Example 2: The areas of two similar triangles are 81 cm² and 49 cm² respectively. If the altitude of the first triangle is 6.3 cm, find the corresponding altitude of the second triangle.

Solution:

- $Area_1/Area_2 = (altitude_1/altitude_2)^2$
- $81/49 = (6.3/altitude_2)^2$
- $9/7 = 6.3/altitude_2$
- $altitude_2 = (6.3 \times 7)/9 = 4.9$ cm

Example 3: Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Solution: Let △ABC ~ △PQR Let AD and PM be medians.

- Since triangles are similar: AB/PQ = BC/QR = AC/PR
- Also, BD/QM = 1/2 BC / 1/2 QR = BC/QR
- In △ABD and △PQM:
    - AB/PQ = BD/QM (proved above)

    - ∠B = ∠Q (similar triangles)
- By SAS, △ABD ~ △PQM
- Therefore, AD/PM = AB/PQ
- Area(△ABC)/Area(△PQR) = $(AB/PQ)^2 = (AD/PM)^2$
- Hence proved.

Example 4: In a right triangle ABC, right-angled at B, BC = 12 cm and AB = 5 cm. A circle is inscribed in the triangle. Find the radius of the circle.

Solution:

- First find AC using Pythagoras: $AC^2 = AB^2 + BC^2$
- $AC^2 = 5^2 + 12^2 = 25 + 144 = 169$
- AC = 13 cm

For inscribed circle in right triangle:

- Radius r = (AB + BC - AC)/2
- r = (5 + 12 - 13)/2
- r = 4/2 = 2 cm

Example 5: Prove that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Solution: Given: △ABC with ∠B = 90° To Prove: $AC^2 = AB^2 + BC^2$

Construction: Draw BD ⊥ AC

Proof:

- In △ADB and △ABC:
    - ∠A = ∠A (common)
    - ∠ADB = ∠ABC = 90°
- By AA, △ADB ~ △ABC
- Therefore, AD/AB = AB/AC
- $AB^2 = AD \times AC$ ... (i)

Similarly, △BDC ~ △ABC

- CD/BC = BC/AC
- $BC^2 = CD \times AC$ ... (ii)

Adding (i) and (ii):

- $AB^2 + BC^2 = AD \times AC + CD \times AC$
- $AB^2 + BC^2 = AC(AD + CD)$
- $AB^2 + BC^2 = AC \times AC$
- $AB^2 + BC^2 = AC^2$ (Hence Proved)

14. IMPORTANT POINTS TO REMEMBER

✓ Two triangles are similar if corresponding angles are equal and corresponding sides are proportional

✓ For proving similarity, AA criterion is most commonly used

✓ Area ratio = $(\text{side ratio})^2$

✓ Perimeter ratio = side ratio

✓ In similar triangles, altitudes, medians, angle bisectors are also in the same ratio as sides

✓ Pythagoras theorem applies only to right triangles

✓ To check if triangle is right-angled, verify if $a^2 + b^2 = c^2$

✓ Scale factor k means sides are in ratio k, areas in ratio k^2

✓ All equilateral triangles are similar

✓ All isosceles right triangles are similar

15. FORMULAS AT A GLANCE

S.No.	Formula/Result	Application
1	△ABC ~ △PQR → AB/PQ = BC/QR = CA/RP	Proportional sides
2	$\text{Area}_1/\text{Area}_2 = (\text{side}_1/\text{side}_2)^2$	Area of similar triangles
3	$\text{Perimeter}_1/\text{Perimeter}_2 = \text{side}_1/\text{side}_2$	Perimeter ratio
4	AD/DB = AE/EC (if DE ∥ BC)	Basic Proportionality
5	$AC^2 = AB^2 + BC^2$ (∠B = 90°)	Pythagoras Theorem
6	$\text{Altitude}_1/\text{Altitude}_2 = \text{side}_1/\text{side}_2$	Altitude ratio
7	$\text{Median}_1/\text{Median}_2 = \text{side}_1/\text{side}_2$	Median ratio

16. QUICK REVISION CHECKLIST

☐ Can you state all four similarity criteria?

☐ Do you know the difference between similarity and congruence?

☐ Can you apply Basic Proportionality Theorem?

☐ Do you remember the area ratio formula?

☐ Can you state and apply Pythagoras Theorem?

☐ Do you know when to use which similarity criterion?

☐ Can you solve problems involving scale factor?

☐ Have you practiced all theorem proofs?

☐ Can you identify similar triangles in complex figures?

☐ Do you know all important formulas by heart?

ALL THE BEST! 📐📚

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