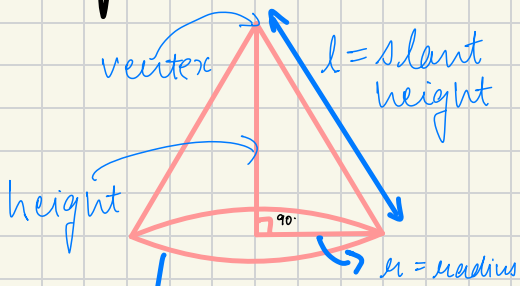
Surface Area & Volume

Area	Surface Area	Volume
valid for 2-D objects circle, square, rectangle	valid for 3-D objects cone, cylinder, sphere, etc. CSA ↙ ↘ TSA	valid for 3-D objects cone, cylinder, sphere etc.

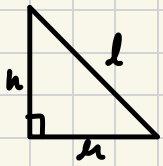
★ Right Circular Cone



By pythagoreas theorem,

$$l^2 = r^2 + h^2$$

$$\boxed{l = \sqrt{r^2 + h^2}}$$



Curved Surface Area = πrl

Total Surface Area = CSA + area of circular base

$= \pi rl + \pi r^2$

$$\boxed{TSA = \pi r (l + r)}$$

Q) The height of the cone is 16cm and its base radius is 12cm. Find the CSA & TSA of the cone. (Use $\pi = 3.14$)

$h = 16\text{ cm}, \quad r = 12\text{ cm}$

$l = \sqrt{r^2 + h^2} = \sqrt{12^2 + 16^2} = \sqrt{144 + 256} = \sqrt{400} = 20$

$\boxed{l = 20\text{ cm}}$

$CSA = \pi r l = 3.14 \times 12 \times 20 = \boxed{753.6\text{ cm}^2}$

$TSA = \pi r (l + r) = 3.14 \times 12 (20 + 12) = 3.14 \times 12 \times 32$

$\boxed{TSA = 1205.76\text{ cm}^2}$

Q) CSA of the cone is 308 cm^2 and its slant height is 14cm. Find the :-

① Radius of the base

$CSA = 308\text{ cm}^2$

$CSA = 308$

$\pi r l = 308$

$\frac{22}{7} \times r \times 14 = 308$

$\boxed{r = 7\text{cm}}$

② TSA of the cone

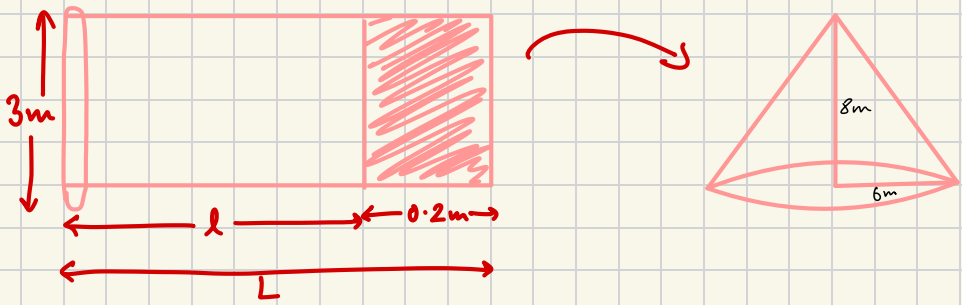
$TSA = \pi r (l + r)$

$= \frac{22}{7} \times 7 \times (14 + 7)$

$= 22 \times 21$

$\boxed{= 462\text{ cm}^2}$

Q) What length of tarpaulin 3m wide will be required to make a conical tent of height 8m & base radius 6m? Assume the extra length of the material that will be required for the stitching margin and wastage in cutting is approx. 20 cm. (Use $\pi = 3.14$)



height of cone = 8m

radius = 6m

$l = \sqrt{r^2 + h^2} = \sqrt{36 + 64} = \sqrt{100}$

$l = 10m$

CSA of conical tent = area of tarpaulin used

πrl = w x length

$3.14 \times 6 \times 10 = 3x$ length

length = 62.8m

Total length of tarpaulin used = 62.8m + length of waste cutting

$= 62.8 + 0.2m$

$= 63$

Q) The slant height & base diameter of a conical tomb are 25m & 14m. Find the cost of white washing its curved surface at the rate of ₹210 per 100 m^2

Slant height = 25m

Diameter = 14m

$r = \frac{14}{2} = 7m$

$CSA = \pi rl = \frac{22}{7} \times 7 \times 25 = 22 \times 25 = \boxed{550\ m^2}$

Cost of white washing 100 m^2 area = ₹210

Cost of white washing 1 m^2 area = $₹\frac{210}{100} = ₹2.1$

Cost of white washing 550 m^2 area = ₹(550 × 2.1)

= ₹1155

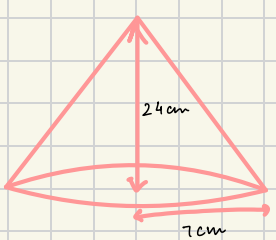
Hence, our cost of white washing the curved surface of conical tomb is ₹1155

Q) A Jocker's Cap is in the form of a right circular cone of the base radius 7cm & height 24cm. Find the area of sheet required to make 10 such caps.

We Know that,

$l^2 = r^2 + h^2$

$l = \sqrt{7^2 + 24^2} = \boxed{250\ m^2}$



Area of sheet required to make 1 cap $= \pi rl$

$= \frac{22}{7} \times 7 \times 25$

$= 550 \text{ cm}^2$

Area of sheet req. to make 10 Joker's cap $= (550 \times 10) \text{ cm}^2$

$= 5500 \text{ cm}^2$

★ Volume of cone

$$\frac{1}{3} \pi r^2 h$$

Q) The volume of a right circular cone is 9856 cm^3. If the diameter of the base is 28cm, find

i) height of cone
ii) slant height of cone
iii) CSA of cone

volume $= 9856 \text{ cm}^3$, diameter $= 28 \text{ cm} \Rightarrow r = 14 \text{ cm}$

$\frac{1}{3} \pi r^2 h = 9856$

$h = \frac{9856 \times 3 \times 7}{22 \times 14 \times 14} = 48 \text{ cm}$

$l^2 = h^2 + r^2 = 48^2 + 14^2 \Rightarrow 2,304 + 196 = 500 \text{ cm}$

$CSA = \pi r l \Rightarrow \frac{22}{7} \times 14^2 \times 500 \Rightarrow 22,000$

Q) If the volume of two cones are in the ratio 1:4 & their diameters are the ratio 4:5, then what is the ratio of their heights?

Let the dimensions of cone 1 be $V_1, r_1, h_1, CSA_1, TSA_1$

Let the dimensions of cone 2 be $V_2, r_2, h_2, CSA_2, TSA_2$

A.T.Q,

$$\frac{d_1}{d_2} = \frac{4}{5} \Rightarrow \frac{2r_1}{2r_2} \Rightarrow \frac{4}{5} \Rightarrow \frac{r_1}{r_2} = \frac{4}{5} \Rightarrow 4:5$$

Now,

$$\frac{V_1}{V_2} = \frac{1}{4} \Rightarrow \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h} \Rightarrow \frac{1}{4} \Rightarrow \frac{h_1}{h_2} = \frac{1}{4} \times \frac{r_2^2}{r_1^2}$$

$$\boxed{\frac{h_1}{h_2} = \frac{25}{64}}$$

Q) A conical pit of top diameter 3.5m is 12m deep. What is the capacity in kiloleters?

$d = 3.5m$, $r = \frac{3.5}{2}$ m

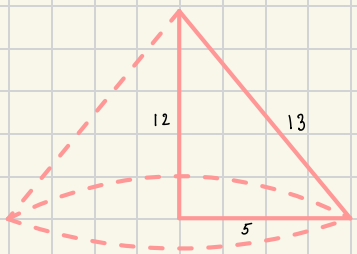
$$V = \frac{1}{3}\pi r^2 h \Rightarrow \frac{1}{3} \times \frac{22}{7} \times \frac{35}{20} \times \frac{35}{20} \times 12 \ m^3$$

$$\Rightarrow \frac{11}{24} \times 7 \times 12 \ m^3 \Rightarrow 38.5 \ m^3$$

$1m^3 = 1KL$

$$\boxed{V = 38.5 \ kL}$$

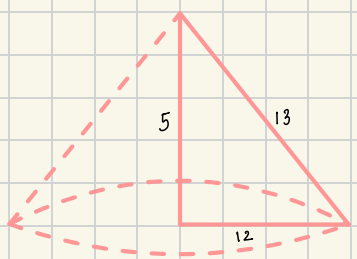
Q) A right ΔABC with its sides 5cm, 12cm & 13 cm is revolved about the side 12cm. Find the volume of the solid so formed. If the ΔABC is revolved about side 5cm, then find the volume of the solid obtained. Find the ratio of the volumes of the two solids obtained.



Let the volume be V_1

$$V_1 = \frac{1}{3}\pi r_1^2 h_1$$

$$= \frac{1}{3}\pi \times 5 \times 5 \times 2$$

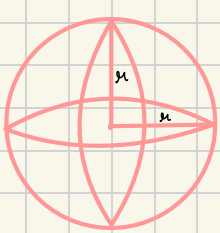


$$V_2 = \frac{1}{3}\pi r_2^2 h_2$$

$$= \frac{1}{3}\pi \times 12 \times 12 \times 5$$

Required ratio $\Rightarrow \frac{V_1}{V_2} = \frac{1/3\,\pi \times 25 \times 12}{1/3\,\pi \times 12 \times 12 \times 5} \Rightarrow \frac{5}{12}$

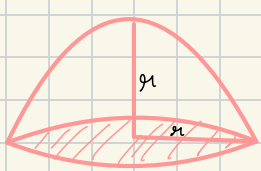
★ SPHERE & HEMISPHERE



CSA : $4\pi r^2$

TSA : $CSA = 4\pi r^2$

Volume : $\frac{4}{3}\pi r^3$



CSA : $2\pi r^2$

TSA : CSA + Base Area

$= 2\pi r^2 + \pi r^2 = 3\pi r^2$

Volume : $\frac{2}{3}\pi r^3$

Q) The diameter of the moon is approx. $\frac{1}{4}$th of the diameter of the earth. Find the ratio of their surface areas and their volumes.

Let the diameter & radius of the moon be d_m and r_m. Let the diameter & radius of earth be d_e & r_e.

AT Q,

$$d_m = \frac{1}{4} d_e$$

$$2r_m = \frac{1}{4} \times 2r_e$$

$$r_m = \frac{r_e}{4}$$

$$\boxed{r_e = 4r_m}$$

$$\frac{(TSA)_{moon}}{(TSA)_{earth}} = \frac{4\pi r_m^2}{4\pi r_e^2} = \frac{r_m^2}{r_e^2} = \frac{r_m^2}{(4r_m)^2} = \frac{r_m^2}{16 r_m^2} \Rightarrow \frac{1}{16}$$

$$\boxed{TSA_{moon} : TSA_{earth} = 1:16}$$

$$\frac{V_{moon}}{V_{earth}} = \frac{4/3\,\pi r_m^3}{4/3\,\pi r_e^2} \Rightarrow \frac{r_m^3}{r_e^3} \Rightarrow \frac{r_m^3}{(4r_m)^3} \Rightarrow \frac{r_m^3}{64 r_m^3} = \boxed{\frac{1}{64}}$$

$$\boxed{V_{moon} : V_{earth} = 1:64}$$

Q) The surface area of all sphere of radius 5cm is 5 times the area of the curved surface of a cone of radius 4 cm. Find the height of the cone.

Radius of sphere = $r_S = 5$cm
Radius of cone = $r_C = 4$ cm

A.T.Q,

TSA of sphere = 5 x CSA of cone

$$4\pi r_S^2 = 5 \times \pi r_C l$$

$$4 \times 5 \times 5 = 5 \times 4 \times l$$

$$\boxed{l = 5}$$

$l = r_C^2 + h^2 \Rightarrow 25 = 16 + h^2$

$h^2 = 9 \rightarrow \boxed{h = 3\text{cm}}$

Q) The radius of a hemi-spherical balloon increase from 7cm to 14 cm as air is being pumped into it. Find the ratios of the surface areas of the balloon in the two cases.

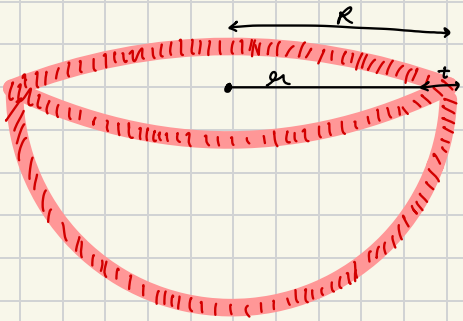
In Case 1, Let $r_1 = 7$cm

In Case 2, Let $r_2 = 14$ cm

A.T.Q,

$$\frac{TSA_1}{TSA_2} = \frac{3\pi r_1^2}{3\pi r_2^2} \Rightarrow \left(\frac{7}{14}\right)^2 \Rightarrow \frac{1}{4}$$

Q) The hemi-spherical bowl is made of steal, 0·25 cm thick. The inner radius of the bowl is 5 cm. Find the outer curved surface area of the bowl.



Outer radius $(R) = r + t$

$R = 5\text{cm} + 0{\cdot}25\text{cm}$

$R = 5{\cdot}25\text{ cm}$

Outer Curved Surface area

$= 2\pi R^2$

$= 2 \times \frac{22}{7} \times (5{\cdot}25)^2$

$= 173\text{ cm}^2$

Q) A right circular cylinder just encloses a sphere of radius r as shown in figure.

Find the:

① Surface area of the sphere
② CSA of cylinder
③ Ratio of area obtained in ① & ②

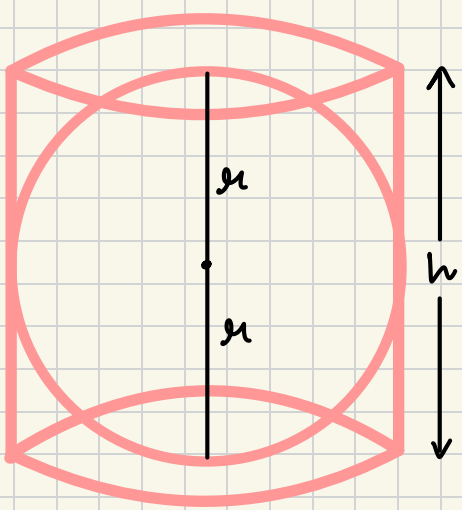
① Surface area of sphere

→ $4\pi r^2$

② CSA of cylinder

→ $2\pi rh = 2\pi r(2r)$

$= 4\pi r^2$



③ Ratio of area

→ $\frac{4\pi r^2}{4\pi r^2} = 1$

Q) Find the volume of sphere, whose surface area is 154 cm^2

Volume of sphere $= \frac{4}{3}\pi r^2 = \frac{4}{3} \times \frac{22}{7} \times r^3$ — (1)

Surface area of sphere $= 154\ cm^2$

$$4\pi r^2 = 154$$
$$4 \times \frac{22}{7} \times r^2 = 154$$
$$r^2 = \frac{7 \times 7}{4}$$
$$r = \frac{7}{2} = 3.5\ cm \quad — (2)$$

Put (2) in (1),

Volume of sphere $= \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}$

$= \boxed{179.6\ cm^3}$

Q) The vol. of two spheres are in the ratio 64 : 27

$$\frac{V_1}{V_2} = \frac{64}{27} \longrightarrow \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27} \longrightarrow \frac{r_1^3}{r_2^3} = \frac{64}{27}$$

So, $\boxed{\frac{r_1}{r_2} = \frac{4}{3}}$ — (1)

Required Ans. $\Rightarrow$ $(T.S.A)_2 - (T.S.A)_1$

$= 4\pi r_1^2 - 4\pi r_2^2$

$= 4\pi (r_1^2 - r_2^2)$

A.T.Q, $r_1 + r_2 = 7$

$\boxed{r_1 = 7 - r_2}$ — (2)

Put eqn. (2) in (1)

$$\frac{7 - r_2}{r_2} = \frac{4}{3}$$

$21 - 3r_2 = 4r_2$

$7r_2 = 21$

$\boxed{r_2 = 3}$

$r_1 = 7 - r_2$

$\boxed{r_1 = 7 - 3 = 4}$

Reqd. Ans. $\Rightarrow 4\pi (r_1^2 - r_2^2) \rightarrow 4\pi (4^2 - 3^2)$

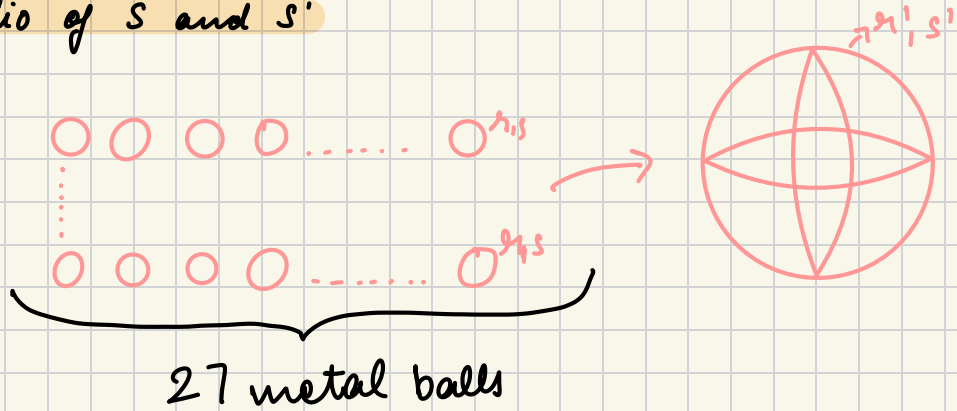
$\Rightarrow 4 \times \frac{22}{7} \times (16 - 9)$

$\Rightarrow 4 \times \frac{22}{7} \times 7$

$= \boxed{88 \text{ cm}^2}$

Q) Twenty seven solid spheres, each of radius r and surface area S are melted to form a sphere area S'. Find the

(i) Radius 'r' of the new sphere
(ii) Ratio of S and S'



Volume of big sphere = Vol. of 27 small sphere

$$\frac{4}{3}\pi (r')^3 = \text{Vol. of 1 small sphere} \times 27$$

$$\frac{4}{3}\pi (r')^3 = \frac{4}{3}\pi r^3 \times 27$$

$$\boxed{r' = 3r} \quad - (1)$$

Required ratio :

$$\frac{S}{S'} = \frac{4\pi r^2}{4\pi (r')^2} = \frac{r^2}{(r')^2} \quad - (2)$$

Put eqn. (1) in eqn. (2)

$$\frac{S}{S'} = \frac{r^2}{(3r)^2} \Rightarrow \frac{r^2}{9r^2} = \frac{1}{9} \longrightarrow \boxed{S : S' = 1 : 9}$$

PRACTICE QUESTIONS

Q) The paint in a certain container is sufficient to paint an area equal to 9.375 sq. m. How many bricks of dimensions 22.5 cm × 10 cm × 7.5 cm can be painted out of this container?

Q) The curved surface area of a right circular cylinder of height 14 cm is 88 cm^2. Find the diameter of the base of the cylinder.

Q) The height of a cone is 16 cm and its base radius is 12 cm. Find the curved surface area of the cone.

Q) A cone is 8.4 cm high and the radius of its base is 2.1 cm. It is melted and recast into a sphere. Find radius of the sphere.

Q) If the volume of a sphere is numerically equally to its surface area, then find the diameter of the sphere.

Q) A spherical ball is divided into two equal halves. If the curved surface area of each half is 56.57 cm ? Find the volume of the spherical ball. [use $\pi = 3.14$]

Q) A right angled ΔABC with sides 3 cm, 4 cm & 5 cm is revolved about the fixed side of 4 cm. Find the volume of the solid generated. Also, find the total surface area of the solid.

Q) How many meters of 5 m wide cloth will be required to make a conical tent, the radius of whose base is 3.5 m & height is 12 m?

Q) A semi-circular sheet of metal of radius 14 cm is bent to form an open conical cup. Find the capacity of the cup

Q) Using clay, Anant made a right circular cone of height 48 cm & base radius 12 cm. Varsha reshapes it in the form of a sphere. Find the radius and curved surface area of the sphere so formed