StudyByU

SOME APPLICATIONS OF TRIGONOMETRY

1. INTRODUCTION

1.1 What is this Chapter About?

In this chapter, we learn how to use trigonometry in real-life situations to find:

- **Heights of buildings, towers, mountains**
- **Distances between objects**
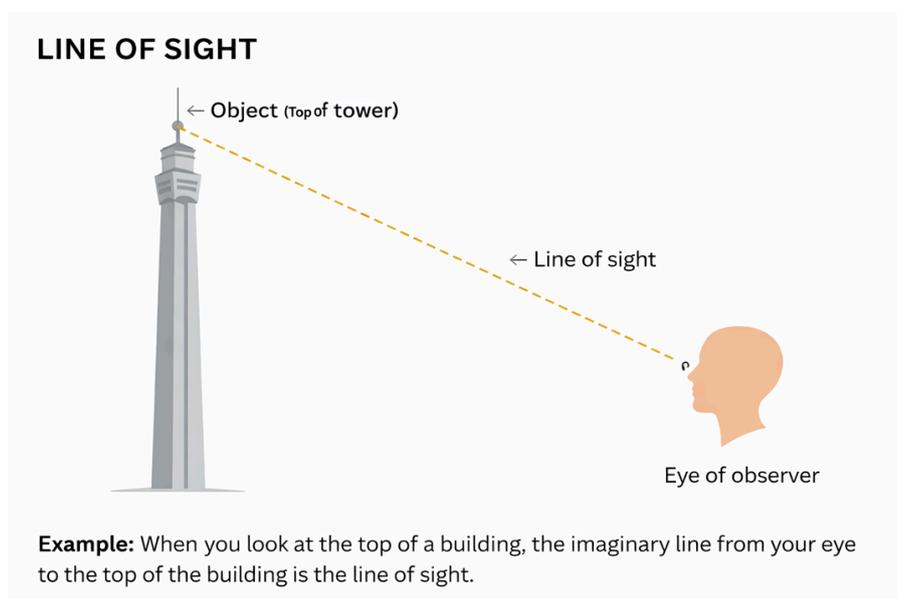- **Without actually measuring them!**

Real-Life Applications:

- **Finding height of a building**
- **Distance of a ship from shore**
- **Height of a mountain**
- **Distance across a river**
- **Height of a flying kite**

2. LINE OF SIGHT

2.1 Definition

Line of Sight: **The line drawn from the eye of an observer to the point in the object viewed by the observer.**

In simple words: **It's the straight line along which you look at an object.**

LINE OF SIGHT

← Object (Top of tower)

← Line of sight

Eye of observer

Example: When you look at the top of a building, the imaginary line from your eye to the top of the building is the line of sight.
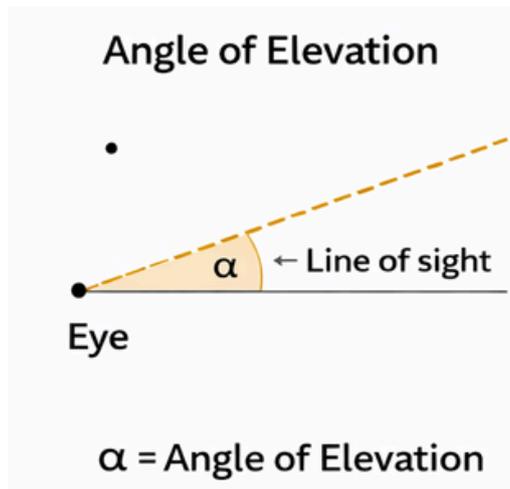
3. ANGLE OF ELEVATION

3.1 Definition

Angle of Elevation: The angle formed by the line of sight with the horizontal when the object is above the horizontal level.

In simple words: When you look UP at something, the angle your line of sight makes with the horizontal is the angle of elevation.

Key Points:

- Object is above eye level
- You raise your head to look at it
- Angle is measured from horizontal upward

Examples:

- Looking at top of a tower
- Looking at a flying bird
- Looking at top of a mountain
- Looking at sun in the sky

3.2 Important Note

The angle of elevation is always measured from the horizontal line, NOT from the ground or any other surface.

4. ANGLE OF DEPRESSION

4.1 Definition

Angle of Depression: The angle formed by the line of sight with the horizontal when the object is below the horizontal level.

In simple words: **When you look DOWN at something, the angle your line of sight makes with the horizontal is the angle of depression.**
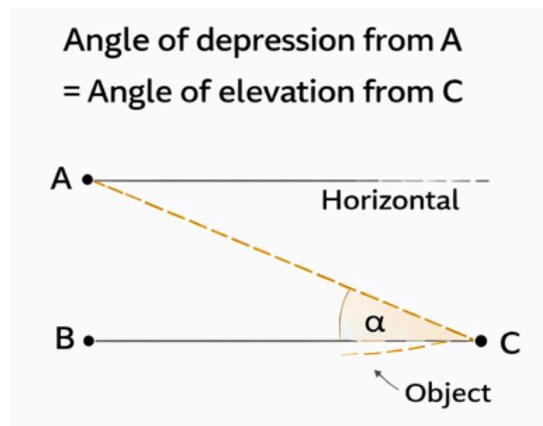
Angle of Depression

Eye

β ← Horizontal

Horizontal

β = Angle of Depression
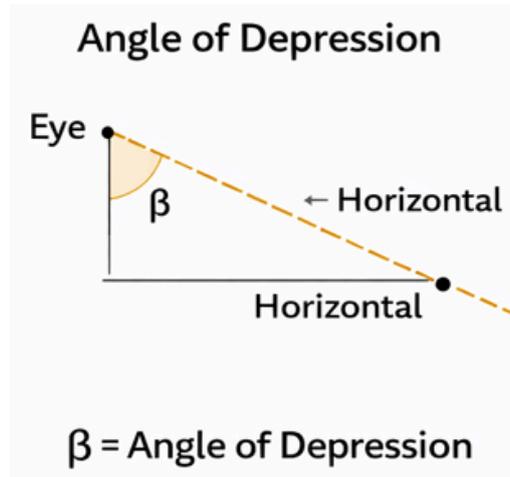
Key Points:

- **Object is below eye level**
- **You lower your head to look at it**
- **Angle is measured from horizontal downward**

Examples:

- **Looking down from a building at a car**
- **Looking from a cliff at a boat in the sea**
- **Looking from a balcony at the ground**
- **Pilot looking down at the runway**

4.2 Important Property

Angle of elevation = Angle of depression (when measured from two different points on the same vertical line)

Angle of depression from A
= Angle of elevation from C

A •

Horizontal

B • α
                                                    • C

Object

Angle of depression from A = Angle of elevation from C
(Alternate angles, parallel lines)

5. SOLVING HEIGHT AND DISTANCE PROBLEMS

5.1 General Steps

Step 1: DRAW A DIAGRAM

- Draw a clear figure
- Mark all known values
- Mark unknown value with a variable (h, x, etc.)

Step 2: IDENTIFY THE RIGHT TRIANGLE

- Most problems involve right-angled triangles
- Identify the right angle

Step 3: CHOOSE TRIGONOMETRIC RATIO

- Based on given and required values
- tan θ is most commonly used (relates height and distance)

Step 4: FORM EQUATION

- Write the trigonometric equation
- Substitute known values

Step 5: SOLVE

- Calculate the unknown
- Give answer with proper units

5.2 Which Trigonometric Ratio to Use?

Most Common: tan θ = Perpendicular/Base

Why tan θ is popular:

- Directly relates height (perpendicular) and distance (base)
- Most problems ask for height or distance

When to use sin θ:

- When hypotenuse is given
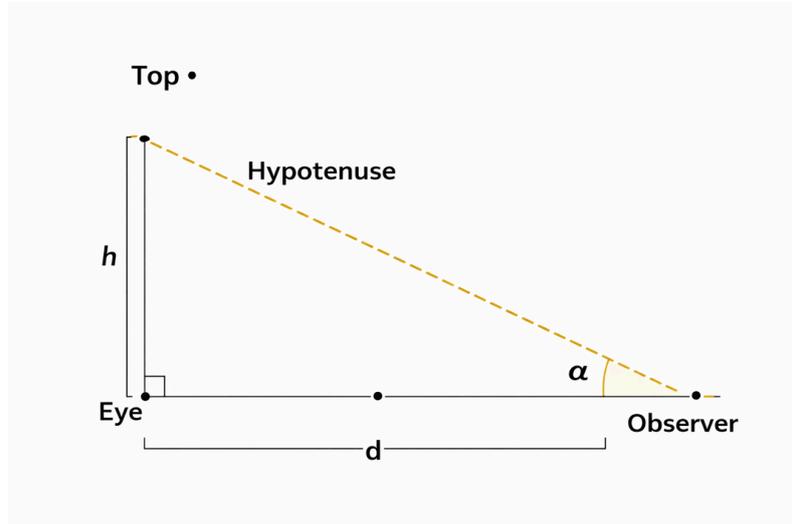- sin θ = Perpendicular/Hypotenuse

When to use cos θ:

- When hypotenuse is given
- cos θ = Base/Hypotenuse

6. STANDARD PROBLEM TYPES

6.1 Type 1: Finding Height of an Object

Problem Setup:

- Observer at some distance from object
- Angle of elevation given
- Find height

Formula:

$\tan \alpha = h/d$

$\therefore h = d \times \tan \alpha$

Example: From a point 30 m away from a tower, the angle of elevation of its top is 60°. Find the height.
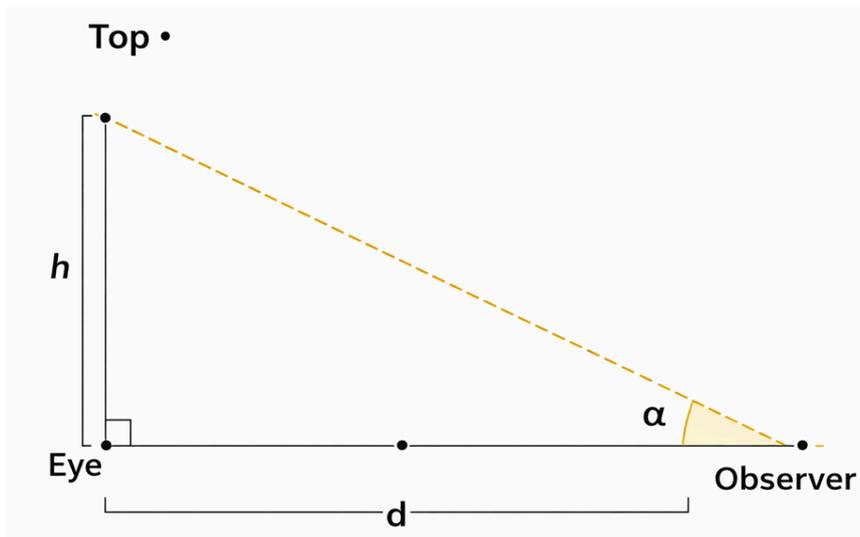
$\tan 60° = h/30$

$\sqrt{3} = h/30$

$h = 30\sqrt{3} = 51.96$ m

6.2 Type 2: Finding Distance

Problem Setup:

- Height of object given
- Angle of elevation given
- Find distance

Formula:

tan α = h/d

∴ d = h/tan α = h × cot α

Example: A tower is 50 m high. From a point on the ground, the angle of elevation is 45°. Find the distance.
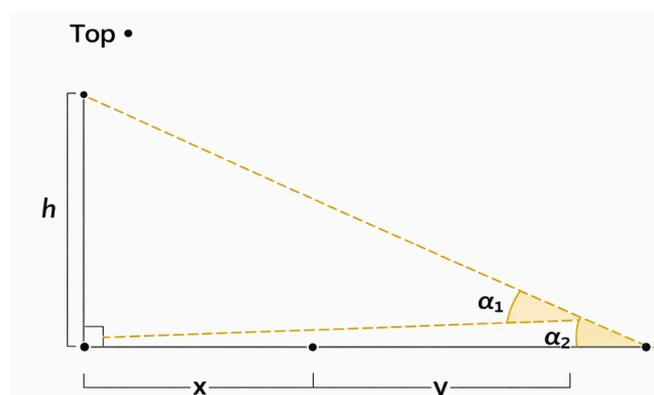
tan 45° = 50/d

1 = 50/d

d = 50 m

6.3 Type 3: Two Angles of Elevation

Problem Setup:

- **Two observations from two different points**
- **Two angles given**
- **Find height**

Approach:

- **Form two equations using both angles**
- **Solve simultaneously**

Example: From a point, angle of elevation is 30°. On moving 20 m closer, it becomes 60°. Find height.

From farther point:

tan 30° = h/(x + 20)

1/√3 = h/(x + 20)

x + 20 = h√3 ... (1)

From nearer point:

tan 60° = h/x

√3 = h/x

x = h/√3 ... (2)

Substitute (2) in (1):

h/√3 + 20 = h√3

20 = h√3 - h/√3

20 = h(√3 - 1/√3)

20 = h(3 - 1)/√3

20 = 2h/√3

h = 10√3 ≈ 17.32 m

6.4 Type 4: Angle of Depression

Problem Setup:

- **Observer at height**
- **Looking down at object**
- **Angle of depression given**
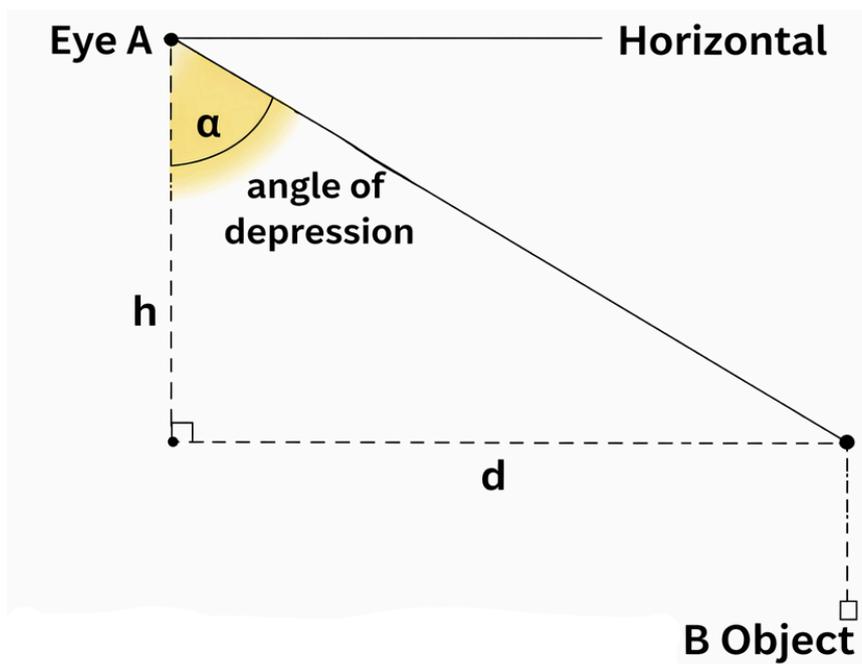
Important: Angle of elevation from B = Angle of depression from A = α

Formula:

tan α = h/d

Example: From top of 75 m building, angle of depression of a car is 30°. Find distance.

tan 30° = 75/d

1/√3 = 75/d

d = 75√3 ≈ 129.9 m

6.5 Type 5: Combined Elevation and Depression

Problem Setup:

- **From top of one building, see:**
  - ○ **Top of another building (elevation)**
  - ○ **Bottom of another building (depression)**
- **Find height of second building**

Approach:

- **Use angle of elevation for upper part**
- **Use angle of depression for lower part (or distance)**
- **Add/subtract as needed**

7. IMPORTANT FORMULAS

7.1 Basic Trigonometric Ratios

tan θ = Perpendicular/Base = Height/Distance

sin θ = Perpendicular/Hypotenuse

cos θ = Base/Hypotenuse

7.2 Standard Angle Values

MUST MEMORIZE:

Angle	30°	45°	60°
tan	$1/\sqrt{3}$	1	$\sqrt{3}$
sin	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$
cos	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2

Common approximations:

$\sqrt{2} \approx 1.414$

$\sqrt{3} \approx 1.732$

7.3 Useful Relations

cot θ = 1/tan θ

If angle of elevation = θ, then:

Height = Distance × tan θ

Distance = Height × cot θ = Height/tan θ

8. SOLVED EXAMPLES

Example 1: Basic Elevation

Question: A circus artist is climbing a 20 m long rope, tightly stretched from top of vertical pole to ground. If angle with ground is 30°, find height of pole.

Solution:

Given:

Rope (hypotenuse) = 20 m

Angle with ground = 30°

Using sin θ = Perpendicular/Hypotenuse

sin 30° = h/20

1/2 = h/20

h = 10 m

Answer: Height of pole = 10 m

Example 2: Finding Distance

Question: A tree breaks due to storm and bent touches ground 12 m from base. Top makes 30° with ground. Find height of tree.

Solution:

Bent part makes angle 30° with ground

In triangle:

$\tan 30° = h_1/12$

$1/\sqrt{3} = h_1/12$

$h_1 = 12/\sqrt{3} = 4\sqrt{3}$ m

Using Pythagoras in bent part:

Bent length² = 12² + h_1^2

= 144 + 48

= 192

Bent length = $8\sqrt{3}$ m

Total height = h_1 + Bent length = $4\sqrt{3} + 8\sqrt{3} = 12\sqrt{3} \approx 20.78$ m

Answer: Height = $12\sqrt{3}$ m ≈ 20.78 m

Example 3: Two Positions

Question: From 60 m away from building, angle of elevation is 30°. Moving 30 m closer, find new angle.

Solution:

First position (60 m away):

tan 30° = h/60

h = 60/√3 = 20√3 m

Second position (30 m away):

tan α = h/30 = (20√3)/30 = (2√3)/3

α = tan⁻¹(2√3/3) ≈ 49.1°

Or recognize: 2√3/3 = 2/√3 × √3/√3 = 2√3/3

This is close to tan 60° ≈ 1.732

Actually: tan α ≈ 1.155, so α ≈ 49°

Answer: New angle ≈ 49°

9. IMPORTANT POINTS TO REMEMBER

✓ **Draw a diagram first - Visualization is key**

✓ **Mark all given values - Don't miss any information**

✓ **Angle of elevation = Looking UP**

✓ **Angle of depression = Looking DOWN**

✓ **tan θ is most useful - Relates height and distance directly**

✓ **Angle of depression from A = Angle of elevation from B (alternate angles)**

✓ **Use standard values - 30°, 45°, 60°**

✓ **Units matter - Always mention units in answer**

10. QUICK REVISION POINTS

Summary of Key Concepts

1. Line of Sight

- **Line from eye to object**

2. Angle of Elevation

- **Angle when looking UP**
- **From horizontal upward**

3. Angle of Depression

- **Angle when looking DOWN**
- **From horizontal downward**

4. Most Used Formula

tan θ = Height/Distance

5. Standard Values to Memorize

tan 30° = 1/√3

tan 45° = 1

tan 60° = √3

6. Key Property

- **Angle of depression from A = Angle of elevation from B**

7. Problem Types

- **Find height**
- **Find distance**
- **Two positions**
- **Depression from height**
- **Combined problems**

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