StudyByU

AREAS RELATED TO CIRCLES

1. INTRODUCTION

Area Related to Circles deals with finding areas and perimeters of plane figures that are combinations of circles, sectors, segments, and other shapes.

2. BASIC FORMULAS - CIRCLE

Circle - Definitions

Circle: The collection of all points in a plane which are at a fixed distance from a fixed point.

Key Terms:

- Centre: The fixed point (O)
- Radius: Fixed distance from centre to any point on circle (r)
- Diameter: Line segment passing through centre with endpoints on circle (d = 2r)
- Circumference: Perimeter/boundary of the circle
- Chord: Line segment joining any two points on the circle
- Arc: A piece of the circumference
- Semicircle: Half of a circle
- Quadrant: One-fourth of a circle

Fundamental Formulas

1. Circumference (Perimeter) of Circle:

$C = 2\pi r$ or $C = \pi d$

Where:

- r = radius
- d = diameter = $2r$
- $\pi = 22/7$ or 3.14 (approx)

2. Area of Circle:

$A = \pi r^2$

Example: Find the circumference and area of a circle with radius 7 cm.

Solution:

- Circumference = $2\pi r = 2 \times (22/7) \times 7 = 44$ cm
- Area = $\pi r^2 = (22/7) \times 7^2 = (22/7) \times 49 = 154$ cm²

3. SECTOR OF A CIRCLE

Definition

Sector: The region enclosed by two radii and the corresponding arc.

Types:

- Minor Sector: Smaller region (angle < 180°)
- Major Sector: Larger region (angle > 180°)
- Semicircle: When angle = 180°

Central Angle (θ): Angle subtended by the arc at the centre.

Sector Formulas

When angle θ is in degrees:

1. Length of Arc:

$l = (\theta/360°) \times 2\pi r$

2. Area of Sector:

$A = (\theta/360°) \times \pi r^2$

3. Perimeter of Sector:

$P = 2r + l = 2r + (\theta/360°) \times 2\pi r$

When angle θ is in radians:

1. Length of Arc:

$l = r\theta$

2. Area of Sector:

$A = (1/2) \times r^2\theta$

Note: In Class 10 CBSE, primarily degree measure is used.

Special Cases

Semicircle (θ = 180°):

- Area = $(1/2)\pi r^2$

- Perimeter = $\pi r + 2r = r(\pi + 2)$

Quadrant ($\theta = 90°$):

- Area = $(1/4)\pi r^2$
- Perimeter = $2r + (1/4) \times 2\pi r = r(2 + \pi/2)$

Example 1: Find the area and perimeter of a sector of a circle with radius 7 cm and central angle 60°.

Solution:

- Area of sector = $(\theta/360°) \times \pi r^2$
- $= (60°/360°) \times (22/7) \times 7^2$
- $= (1/6) \times (22/7) \times 49$
- $= (22 \times 49)/(7 \times 6)$
- $= 1078/42$
- $= 77/3\ cm^2 = 25.67\ cm^2$
- Arc length = $(\theta/360°) \times 2\pi r$
- $= (60°/360°) \times 2 \times (22/7) \times 7$
- $= (1/6) \times 44$
- $= 22/3$ cm = 7.33 cm
- Perimeter = 2r + arc length
- $= 2(7) + 22/3$
- $= 14 + 22/3$
- $= (42 + 22)/3$
- $= 64/3$ cm = 21.33 cm

4. SEGMENT OF A CIRCLE

Definition

Segment: The region enclosed by a chord and the corresponding arc.

Types:

- Minor Segment: Smaller region
- Major Segment: Larger region
- Semicircular region: When chord is a diameter

Segment Formulas

Area of Minor Segment:

A = Area of sector − Area of triangle

$A = (\theta/360°) \times \pi r^2 - (1/2) \times r^2 \times \sin\theta$

OR

$A = \pi r^2(\theta/360°) - r^2\sin(\theta/2)\cos(\theta/2)$

Simplified for common angles:

For θ in degrees: $A = r^2[\pi\theta/360° - \sin\theta/2]$

Area of Major Segment:

A = Area of circle − Area of minor segment

$A = \pi r^2$ − [Area of minor segment]

Special Cases:

Semicircular segment (θ = 180°): Area = $(1/2)\pi r^2$

For θ = 90°: Area of segment = $(\pi r^2/4) - (r^2/2) = r^2(\pi/4 - 1/2) = r^2(\pi - 2)/4$

For θ = 60°: Area of segment = $r^2(\pi/6 - \sqrt{3}/4)$

For θ = 120°: Area of segment = $r^2(\pi/3 + \sqrt{3}/4)$

Example 2: Find the area of the segment of a circle of radius 14 cm, if the angle of the sector is 60°.

Solution: Area of sector = $(60°/360°) \times \pi r^2 = (1/6) \times (22/7) \times 14^2 = (1/6) \times (22/7) \times 196 = (22 \times 196)/(6 \times 7) = 4312/42 = 308/3$ cm²

For △AOB where ∠AOB = 60°: Area of triangle = $(1/2) \times r \times r \times \sin 60° = (1/2) \times 14 \times 14 \times (\sqrt{3}/2) = (196\sqrt{3})/4 = 49\sqrt{3}$ cm² = 49 × 1.732 = 84.87 cm²

Area of segment = Area of sector − Area of triangle = $308/3 - 49\sqrt{3}$ = 102.67 − 84.87 = 17.8 cm² (approx)

5. ANNULUS (CIRCULAR RING)

Definition

Annulus: The region between two concentric circles (circles with the same centre).

Annulus Formulas

Let:

- R = radius of outer circle
- r = radius of inner circle

Area of Annulus:

$A = \pi R^2 - \pi r^2$

$A = \pi(R^2 - r^2)$

$A = \pi(R + r)(R - r)$

Width of Ring: If width $w = R - r$, and if inner radius r is known: $A = \pi[(r + w)^2 - r^2]$ $A = \pi[r^2 + 2rw + w^2 - r^2]$ $A = \pi(2rw + w^2)$ $A = \pi w(2r + w)$

Example 3: A circular park has a path 7 m wide running around it on the outside. If the radius of the park is 70 m, find the area of the path.

Solution: Inner radius (r) = 70 m Outer radius (R) = 70 + 7 = 77 m

Area of path = $\pi(R^2 - r^2) = (22/7) \times (77^2 - 70^2) = (22/7) \times (5929 - 4900) = (22/7) \times 1029 = (22 \times 1029)/7 = 22638/7 = 3234$ m²

6. COMBINATIONS OF PLANE FIGURES

Common Combinations:

1. Circle inscribed in a square
2. Circle circumscribed about a square
3. Square inscribed in a circle
4. Triangle inscribed in a circle
5. Circle inscribed in a triangle
6. Circles and rectangles
7. Semicircles and quarters on sides of figures

Important Relationships

1. Circle inscribed in a Square: If side of square = a, then:

- Radius of circle = $a/2$
- Area of circle = $\pi(a/2)^2 = \pi a^2/4$
- Area of square = a^2
- Area of shaded region = $a^2 - \pi a^2/4 = a^2(1 - \pi/4) = a^2(4 - \pi)/4$

2. Circle circumscribed about a Square: If side of square = a, then:

- Diagonal of square = $a\sqrt{2}$
- Radius of circle = $a\sqrt{2}/2 = a/\sqrt{2}$
- Area of circle = $\pi(a/\sqrt{2})^2 = \pi a^2/2$
- Area of shaded region = $\pi a^2/2 - a^2 = a^2(\pi/2 - 1) = a^2(\pi - 2)/2$

3. Square inscribed in a Circle: If radius of circle = r, then:

- Diagonal of square = diameter = $2r$
- Side of square = $2r/\sqrt{2} = r\sqrt{2}$
- Area of square = $(r\sqrt{2})^2 = 2r^2$
- Area of circle = πr^2

- Area of shaded region = $\pi r^2 - 2r^2 = r^2(\pi - 2)$

4. Circle inscribed in an Equilateral Triangle: If side of triangle = a, then:

- Radius of inscribed circle (inradius) = $a/(2\sqrt{3})$
- Area of circle = $\pi \times [a/(2\sqrt{3})]^2 = \pi a^2/12$

5. Circle circumscribed about an Equilateral Triangle: If side of triangle = a, then:

- Radius of circumscribed circle (circumradius) = $a/\sqrt{3}$
- Area of circle = $\pi \times (a/\sqrt{3})^2 = \pi a^2/3$

6. Semicircles on sides of a Right Triangle: If right triangle has sides a, b, and hypotenuse c:

Sum of areas of semicircles on two legs = Area of semicircle on hypotenuse

$(1/2)\pi(a/2)^2 + (1/2)\pi(b/2)^2 = (1/2)\pi(c/2)^2$

This follows from Pythagoras theorem: $a^2 + b^2 = c^2$

7. WORKED EXAMPLES

Example 4: A horse is tied to a peg at one corner of a square-shaped grass field of side 15 m by means of a 7 m long rope. Find the area of that part of the field in which the horse can graze.

Solution: The horse can graze a quadrant (quarter circle) of radius 7 m.

Area that horse can graze = $(1/4)\pi r^2 = (1/4) \times (22/7) \times 7^2 = (1/4) \times (22/7) \times 49 = (22 \times 49)/(4 \times 7) = 1078/28 = 38.5\ m^2$

Example 5: A copper wire when bent in the form of a square encloses an area of 121 cm^2. If the same wire is bent into the form of a circle, find the area of the circle.

Solution: Area of square = 121 cm^2 Side of square = $\sqrt{121}$ = 11 cm Perimeter of square = 4 × 11 = 44 cm

This perimeter becomes the circumference of circle: $2\pi r = 44$ $r = 44/(2\pi) = 44/(2 \times 22/7) = 44 \times 7/44 = 7$ cm

Area of circle = $\pi r^2 = (22/7) \times 7^2 = (22/7) \times 49 = 154\ cm^2$

Example 6: In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in the figure. Find the area of the design (shaded region).

Solution: For equilateral triangle inscribed in a circle: Side of triangle = $r\sqrt{3}$ (where r is circumradius) = $32\sqrt{3}$ cm

Area of equilateral triangle = $(\sqrt{3}/4) \times side^2 = (\sqrt{3}/4) \times (32\sqrt{3})^2 = (\sqrt{3}/4) \times 1024 \times 3 = (\sqrt{3}/4) \times 3072 = 768\sqrt{3}\ cm^2$

Area of circle = $\pi r^2 = (22/7) \times 32^2 = (22/7) \times 1024 = 22528/7\ cm^2$

Area of design = Area of circle − Area of triangle = $22528/7 - 768\sqrt{3} = 3218.29 - 1330.18 = 1888.11\ cm^2$ (approx)

Example 7: A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding: (i) Minor segment (ii) Major segment

Solution: $r = 10$ cm, $\theta = 90°$

(i) Minor segment:

Area of sector = $(90°/360°) \times \pi r^2 = (1/4) \times (22/7) \times 10^2 = (1/4) \times (22/7) \times 100 = 2200/28 = 550/7\ cm^2$

Area of triangle AOB (right-angled at O): = $(1/2) \times 10 \times 10 = 50\ cm^2$

Area of minor segment = $550/7 - 50 = (550 - 350)/7 = 200/7 = 28.57\ cm^2$

(ii) Major segment:

Area of circle = $\pi \times 10^2 = (22/7) \times 100 = 2200/7\ cm^2$

Area of major segment = $2200/7 - 200/7 = 2000/7 = 285.71\ cm^2$

Example 8: The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is traveling at a speed of 66 km/h?

Solution: Diameter = 80 cm, Radius = 40 cm

Circumference of wheel = $2\pi r = 2 \times (22/7) \times 40 = 1760/7$ cm = 251.43 cm = 2.5143 m

Speed = 66 km/h = 66000 m/h

Distance covered in 10 minutes = $66000 \times (10/60) = 66000 \times (1/6) = 11000$ m

Number of revolutions = Distance/Circumference = 11000/2.5143 = 4375 revolutions

Example 9: The minute hand of a clock is 14 cm long. Find the area on the face of the clock described by the minute hand between 9 AM and 9:35 AM.

Solution: In 60 minutes, minute hand completes 360° In 35 minutes, angle = $(35/60) \times 360° = 210°$

Area swept = $(\theta/360°) \times \pi r^2 = (210°/360°) \times (22/7) \times 14^2 = (7/12) \times (22/7) \times 196 = (7 \times 22 \times 196)/(12 \times 7) = (22 \times 196)/12 = 4312/12 = 359.33\ cm^2$

Example 10: Four equal circles are described about the four corners of a square so that each touches two of the others. If a side of the square is 14 cm, find the area enclosed between the circumferences of the circles.

Solution: Side of square = 14 cm Radius of each circle = 14/2 = 7 cm

Area of square = $14^2 = 196$ cm^2

Each circle contributes a quadrant inside the square. Area of 4 quadrants = $4 \times (1/4)\pi r^2 = \pi r^2 = (22/7) \times 7^2 = (22/7) \times 49 = 154$ cm^2

Area enclosed = Area of square − Area of 4 quadrants = 196 − 154 = 42 cm^2

8. IMPORTANT FORMULAS AT A GLANCE

Figure	Area Formula	Perimeter/Circumference
Circle	πr^2	$2\pi r$
Semicircle	$(1/2)\pi r^2$	$\pi r + 2r$
Quadrant	$(1/4)\pi r^2$	$(\pi r/2) + 2r$
Sector	$(\theta/360°)\pi r^2$	$2r + (\theta/360°)2\pi r$
Segment	$(\theta/360°)\pi r^2 - (1/2)r^2\sin\theta$	Arc + Chord
Annulus	$\pi(R^2 - r^2)$	$2\pi R + 2\pi r$ (both circles)

9. TYPES OF PROBLEMS

Type 1: Basic Circle Problems

- Finding area and circumference given radius/diameter
- Finding radius given area/circumference

Type 2: Sector Problems

- Finding area of sector given angle and radius
- Finding arc length
- Finding perimeter of sector

Type 3: Segment Problems

- Finding area of minor/major segment
- Problems involving chords

Type 4: Annulus Problems

- Circular paths and rings
- Concentric circles

Type 5: Combination Problems

- Figures inside/outside circles
- Shaded regions
- Multiple circles

Type 6: Application Problems

- Wheels and revolutions
- Clock problems
- Grazing problems
- Design problems

10. SPECIAL TIPS FOR BOARD EXAMS

For 2-3 Mark Questions:

- Write formula clearly
- Show all steps
- Write units in answer
- Use $\pi = 22/7$ unless specified

For 4-5 Mark Questions:

- Draw neat labeled diagram
- Break problem into parts
- Show area of each part
- Final answer highlighted

Time Saving Tips:

- Memorize all formulas
- Practice standard angle calculations
- Learn to identify figure type quickly
- Use shortcuts for common problems

11. IMPORTANT POINTS TO REMEMBER

✓ Learn all basic formulas by heart

✓ For sector: angle is at the centre, not at circumference

✓ Segment = Sector − Triangle

✓ Perimeter of sector includes two radii

✓ In annulus, area = $\pi(R^2 - r^2) = \pi(R+r)(R-r)$

✓ Semicircle perimeter = $\pi r + 2r$ (not just πr)

✓ For shaded regions, identify what to add/subtract

✓ Always draw diagram for combination problems

✓ $\pi = 22/7$

✓ Units must be consistent and mentioned

12. QUICK REVISION CHECKLIST

☐ Area and circumference of circle

☐ Area of semicircle and quadrant

☐ Area and perimeter of sector (all formulas)

☐ Area of segment (minor and major)

☐ Area of annulus/circular ring

☐ Circle in square and square in circle

☐ Combinations involving semicircles

☐ Application problems (wheels, clocks, paths)

☐ All worked examples practiced

☐ Previous year questions solved

13. VALUE OF π

Standard Values:

- Exact: π (leave in terms of π)
- Fractional: 22/7 (preferred in exams)
- Decimal: 3.14 or 3.1416

When to use which:

- Use 22/7 when radius is multiple of 7
- Use 3.14 when specified in question

- **Leave in terms of π when asked for exact value**

ALL THE BEST! ⭕📐✨

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