

CIRCLES

1. INTRODUCTION

1.1 What You Already Know

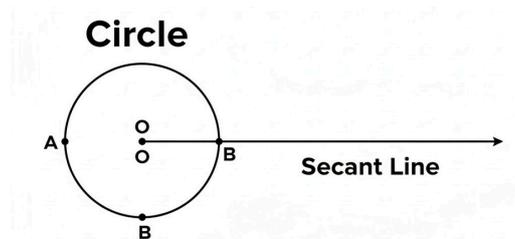
From previous classes, you know:

- A circle is the collection of all points in a plane which are at a constant distance (radius) from a fixed point (centre)
- Chord: Line segment joining two points on circle
- Diameter: Longest chord passing through centre
- Arc: Part of the circle
- Segment: Region between chord and arc

2. BASIC DEFINITIONS

2.1 Secant

Definition: A line which intersects a circle at two distinct points is called a secant of the circle.



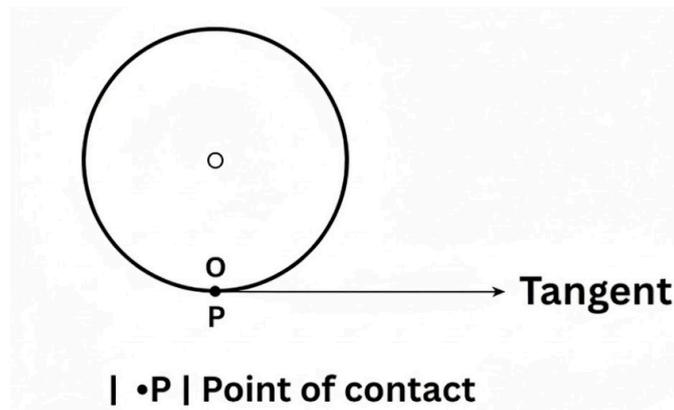
Key Points:

- Cuts circle at 2 points
- Goes through the circle
- Can have unlimited secants

2.2 Tangent

Definition: A line which touches a circle at exactly one point is called a tangent to the circle.

The point where tangent touches the circle is called the point of contact.



Key Points:

- Touches circle at exactly ONE point
- Does not cut through the circle
- Point of contact: Where tangent touches circle

Important: A tangent is a special case of secant when the two points of intersection coincide.

2.3 Relationship Between Secant and Tangent

Secant

Tangent

•——• → • (Two points merge into one)

When the two points of a secant get closer and closer and finally coincide, the secant becomes a tangent.

3. NUMBER OF TANGENTS FROM A POINT

3.1 Case 1: Point Inside the Circle

NO tangent can be drawn from a point inside the circle.

Reason: Any line through P will intersect circle at two points.

Number of tangents: 0 (Zero)

3.2 Case 2: Point ON the Circle

EXACTLY ONE tangent can be drawn from a point on the circle.

Number of tangents: 1 (One)

3.3 Case 3: Point Outside the Circle

EXACTLY TWO tangents can be drawn from a point outside the circle.

Number of tangents: 2 (Two)

Summary Table:

Position of Point	Number of Tangents
Inside circle	0
On circle	1
Outside circle	2

4. THEOREM 1: TANGENT PERPENDICULAR TO RADIUS

★ MOST IMPORTANT THEOREM ★

4.1 Statement

The tangent at any point of a circle is perpendicular to the radius through the point of contact.

In symbols: If OP is radius and PQ is tangent at P , then $OP \perp PQ$

4.2 Proof

Given: A circle with centre O , and a tangent PQ at point P

To Prove: $OP \perp PQ$

Construction: Take any point R on tangent PQ (other than P). Join OR .

Proof:

Step 1: R is a point on tangent PQ

Step 2: R must lie outside the circle (if R was inside, PQ would be a secant)

Step 3: $OR > OP$ (OR is outside, OP is radius)

Step 4: This is true for ALL points on PQ except P

Step 5: So OP is the shortest distance from O to line PQ

Step 6: Shortest distance from a point to a line is the perpendicular

$\therefore OP \perp PQ$

Hence Proved

4.3 Applications

This theorem is used in:

- Finding unknown lengths
- Proving other properties
- Solving numerical problems

5. THEOREM 2: EQUAL TANGENTS FROM EXTERNAL POINT

★ EXTREMELY IMPORTANT THEOREM ★

5.1 Statement

The lengths of tangents drawn from an external point to a circle are equal.

In symbols: If PA and PB are two tangents from external point P, then $PA = PB$

5.2 Proof

Given: A circle with centre O, tangents PA and PB from external point P

To Prove: $PA = PB$

Construction: Join OA, OB, and OP

Proof:

In $\triangle OAP$ and $\triangle OBP$:

1. $OA = OB$ (both radii)
2. $\angle OAP = \angle OBP = 90^\circ$ (tangent \perp radius, Theorem 1)
3. $OP = OP$ (common)

By RHS congruence:

$$\triangle OAP \cong \triangle OBP$$

$$\therefore PA = PB \text{ (CPCT)}$$

Hence Proved

5.3 Additional Results from Proof

From the same proof, we also get:

1. $PA = PB$ (main result)
2. $\angle APO = \angle BPO$ (OP bisects $\angle APB$)
3. $\angle AOP = \angle BOP$ (OP bisects $\angle AOB$)

In summary: OP is the angle bisector of both $\angle APB$ and $\angle AOB$

6. IMPORTANT PROPERTIES

6.1 Properties of Tangents (Summary)

Property 1: Tangent is perpendicular to radius at point of contact

$OP \perp$ tangent (where P is point of contact)

Property 2: Tangents from external point are equal in length

$PA = PB$ (from external point P)

Property 3: Line joining external point to centre bisects angle between tangents

OP bisects $\angle APB$

Property 4: Only ONE tangent at any point on circle

Property 5: Exactly TWO tangents from any external point

6.2 Finding Length of Tangent

Formula: If tangent is drawn from external point P to circle with centre O and radius r, and $OP = d$, then:

Length of tangent = $\sqrt{d^2 - r^2}$

Using Pythagoras Theorem:

In right $\triangle OAP$:

$$OP^2 = OA^2 + PA^2$$

$$d^2 = r^2 + PA^2$$

$$PA^2 = d^2 - r^2$$

$$PA = \sqrt{d^2 - r^2}$$

7. SOLVED EXAMPLES

Example 1: Basic Application

Question: In the figure, if TP and TQ are two tangents to a circle with centre O such that $\angle POQ = 110^\circ$, find $\angle PTQ$.

Solution:

Given: $\angle POQ = 110^\circ$

In quadrilateral OPTQ:

$\angle OPT = 90^\circ$ (tangent \perp radius)

$\angle OQT = 90^\circ$ (tangent \perp radius)

$\angle POQ = 110^\circ$ (given)

Sum of angles in quadrilateral = 360°

$\angle OPT + \angle PTQ + \angle TQO + \angle QOP = 360^\circ$

$90^\circ + \angle PTQ + 90^\circ + 110^\circ = 360^\circ$

$\angle PTQ + 290^\circ = 360^\circ$

$\angle PTQ = 70^\circ$

Answer: $\angle PTQ = 70^\circ$

Example 2: Finding Length

Question: From a point Q, the length of tangent to a circle is 24 cm and distance of Q from centre is 25 cm. Find the radius of the circle.

Solution:

Let radius = r

Distance from centre = d = 25 cm

Length of tangent = 24 cm

Using: $\text{tangent}^2 + \text{radius}^2 = \text{distance}^2$

$$24^2 + r^2 = 25^2$$

$$576 + r^2 = 625$$

$$r^2 = 49$$

$$r = 7 \text{ cm}$$

Answer: Radius = 7 cm

Example 3: Two Tangents

Question: PQ is a tangent to a circle with centre O at point P. If $\triangle OPQ$ is an isosceles triangle, find $\angle POQ$.

Solution:

Given: $\triangle OPQ$ is isosceles

Since $OP \perp PQ$ (tangent perpendicular to radius)

$$\angle OPQ = 90^\circ$$

For isosceles triangle, two sides must be equal.

Since $\angle OPQ = 90^\circ$, OP and PQ cannot be equal.

$$\therefore OP = OQ \text{ or } PQ = OQ$$

If $OP = OQ$ (isosceles with $OP = OQ$):

$$\text{Let } OP = OQ = x$$

$$\angle OPQ = 90^\circ$$

$$\angle OQP = \angle OPQ \text{ is impossible (one is } 90^\circ)$$

So $PQ = OQ$ (isosceles with $PQ = OQ$):

$$\text{Let } \angle POQ = \theta$$

Then $\angle OQP = \theta$ (isosceles triangle)

$$\angle OPQ + \angle POQ + \angle OQP = 180^\circ$$

$$90^\circ + \theta + \theta = 180^\circ$$

$$2\theta = 90^\circ$$

$$\theta = 45^\circ$$

Answer: $\angle POQ = 45^\circ$

8. IMPORTANT RESULTS TO REMEMBER

8.1 In a Right-Angled Triangle with Tangents

If tangents are drawn from external point to a circle, and centre is joined to external point, then:

1. Two right-angled triangles are formed
2. These triangles are congruent (RHS)
3. Tangents are equal
4. Angles at external point are equal

8.2 Quadrilateral Formed

When two tangents are drawn from external point, a quadrilateral is formed with:

- Two right angles (at points of contact)
- Sum of opposite angles = 180°
- Line joining centre to external point is angle bisector

9. QUICK REVISION POINTS

Summary of Key Concepts

1. Tangent:

- Line touching circle at exactly ONE point

2. Point of Contact:

- Point where tangent touches circle

3. Number of Tangents:

- From inside: 0
- From on circle: 1
- From outside: 2

4. Theorem 1:

Tangent \perp Radius at point of contact

$OP \perp PQ$

5. Theorem 2:

Tangents from external point are equal

$PA = PB$

6. Length of Tangent Formula:

$PA = \sqrt{OP^2 - OA^2}$

$$PA = \sqrt{d^2 - r^2}$$

7. Key Property:

OP bisects $\angle APB$ (angle between tangents)

10. PRACTICE CHECKLIST

Before Exam:

- Know both theorems (statement + proof)
- Memorize: Tangent \perp radius
- Memorize: $PA = PB$ from external point
- Practice drawing neat diagrams (20+ times)
- Solve all NCERT Exercise 10.1 questions (4 Q)
- Solve all NCERT Exercise 10.2 questions (13 Q)
- Practice 15 problems on finding lengths
- Practice 10 problems on finding angles
- Practice 10 proof-based questions
- Revise both theorem proofs 5 times each
- Solve previous year questions (last 5 years)
- Time yourself on 10 problems

All the Best for Board Exams! 

