COORDINATE GEOMETRY

1. INTRODUCTION TO COORDINATE GEOMETRY

Coordinate Geometry (also called Cartesian Geometry or Analytical Geometry) is the study of geometry using a coordinate system.

Historical Note: **Developed by French mathematician René Descartes, which is why it's also called Cartesian Geometry.**

Purpose: **To represent geometrical figures algebraically and solve geometrical problems using algebraic methods.**

2. COORDINATE SYSTEM

Cartesian Plane

A plane with two perpendicular lines (axes) is called a Cartesian plane or coordinate plane.

Components:

- **X-axis: Horizontal line**
- **Y-axis: Vertical line**
- **Origin (O): Point of intersection of axes, coordinates (0, 0)**

Coordinates of a Point: An ordered pair (x, y) where:

- **x = x-coordinate (abscissa) - distance from y-axis**
- **y = y-coordinate (ordinate) - distance from x-axis**

Quadrants

The axes divide the plane into four parts called quadrants:

Quadrant	Position	Signs	Example
I	Top-right	(+, +)	(3, 4)
II	Top-left	(−, +)	(−2, 5)
III	Bottom-left	(−, −)	(−3, −2)
IV	Bottom-right	(+, −)	(4, −3)

Important Points:

- Points on x-axis have y-coordinate = 0, like (a, 0)
- Points on y-axis have x-coordinate = 0, like (0, b)
- Origin has coordinates (0, 0)

3. DISTANCE FORMULA

Formula to Find Distance Between Two Points

Distance between points $A(x_1, y_1)$ and $B(x_2, y_2)$:

$AB = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

Derivation: Uses Pythagoras theorem on the right triangle formed.

Special Cases:

1. Distance from origin O(0, 0) to point P(x, y):
   - $OP = \sqrt{(x^2 + y^2)}$
2. Distance between points on x-axis $A(x_1, 0)$ and $B(x_2, 0)$:
   - $AB = |x_2 - x_1|$
3. Distance between points on y-axis $A(0, y_1)$ and $B(0, y_2)$:
   - $AB = |y_2 - y_1|$

Examples:

Example 1: Find the distance between A(3, 4) and B(6, 8).

Solution:

- $x_1 = 3, y_1 = 4, x_2 = 6, y_2 = 8$
- $AB = \sqrt{[(6-3)^2 + (8-4)^2]}$
- $AB = \sqrt{[3^2 + 4^2]}$

- $AB = \sqrt{[9 + 16]}$
- $AB = \sqrt{25} = 5$ units

Example 2: Find the distance between P(−2, 3) and Q(4, −5).

Solution:

- $AB = \sqrt{[(4-(-2))^2 + (-5-3)^2]}$
- $AB = \sqrt{[(6)^2 + (-8)^2]}$
- $AB = \sqrt{[36 + 64]}$
- $AB = \sqrt{100} = 10$ units

4. SECTION FORMULA

Internal Division

When a point P divides the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio m:n:

$P(x, y) = ((mx_2 + nx_1)/(m+n), (my_2 + ny_1)/(m+n))$

Example: Find the coordinates of point P which divides the line segment joining A(2, 3) and B(8, 9) in the ratio 2:1 internally.

Solution:

- $m = 2, n = 1$
- $x_1 = 2, y_1 = 3, x_2 = 8, y_2 = 9$
- $x = (2\times8 + 1\times2)/(2+1) = (16+2)/3 = 18/3 = 6$
- $y = (2\times9 + 1\times3)/(2+1) = (18+3)/3 = 21/3 = 7$
- P = (6, 7)

External Division

When a point P divides the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ externally in the ratio m:n:

$P(x, y) = ((mx_2 - nx_1)/(m-n), (my_2 - ny_1)/(m-n))$

Note: In CBSE Class 10, external division is not in the current syllabus, but internal division is very important.

5. MIDPOINT FORMULA

Special case of Section Formula when m:n = 1:1

Midpoint of line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$:

M = $((x_1+x_2)/2, (y_1+y_2)/2)$

Simple Rule: Average of x-coordinates and average of y-coordinates.

Example: Find the midpoint of the line segment joining A(3, 5) and B(7, 9).

Solution:

- M = ((3+7)/2, (5+9)/2)
- M = (10/2, 14/2)
- M = (5, 7)

6. AREA OF TRIANGLE

Formula for Area of Triangle with Given Vertices

Area of triangle with vertices $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$:

Area = $\frac{1}{2} |x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)|$

Alternative Formula (Determinant Form):

Area = $\frac{1}{2} |x_1y_2 + x_2y_3 + x_3y_1 - x_1y_3 - x_2y_1 - x_3y_2|$

Important Points:

- Area is always positive (use absolute value)
- If area = 0, the three points are collinear
- Unit of area = $(\text{unit of coordinates})^2$

Example: Find the area of triangle with vertices A(1, 2), B(4, 6), C(6, 2).

Solution:

- Area = 1/2 |1(6−2) + 4(2−2) + 6(2−6)|
- = 1/2 |1(4) + 4(0) + 6(−4)|
- = 1/2 |4 + 0 − 24|
- = 1/2 |−20|
- = 1/2 × 20
- = 10 square units

7. CONDITION FOR COLLINEARITY

Three points A, B, C are collinear if and only if:

Area of △ABC = 0

OR

$x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2) = 0$

Example: Check if points A(1, 2), B(3, 6), C(5, 10) are collinear.

Solution:

- Area = 1/2 |1(6−10) + 3(10−2) + 5(2−6)|
- = 1/2 |1(−4) + 3(8) + 5(−4)|
- = 1/2 |−4 + 24 − 20|
- = 1/2 |0|
- = 0
- Since area = 0, points are collinear

8. IMPORTANT APPLICATIONS

Application 1: Finding Unknown Coordinate

When one coordinate is unknown and a condition (like distance or area) is given.

Example: If the distance between points (3, y) and (7, 5) is 5 units, find y.

Solution:

- $\sqrt{[(7-3)^2 + (5-y)^2]} = 5$
- $\sqrt{[16 + (5-y)^2]} = 5$
- $16 + (5-y)^2 = 25$
- $(5-y)^2 = 9$
- $5-y = \pm 3$
- y = 5−3 = 2 or y = 5+3 = 8
- y = 2 or y = 8

Application 2: Finding Vertices of Geometric Figures

For Rectangle/Square:

- Use distance formula to verify equal sides
- Use Pythagoras to verify right angles

For Rhombus:

- All four sides equal (use distance formula)
- Diagonals bisect each other (use midpoint formula)

For Parallelogram:

- Opposite sides equal
- Diagonals bisect each other

Application 3: Finding Centroid of Triangle

Centroid is the point of intersection of medians.

Formula for Centroid G:

$G = ((x_1+x_2+x_3)/3, (y_1+y_2+y_3)/3)$

Simple Rule: Average of all three x-coordinates and all three y-coordinates.

Example: Find the centroid of triangle with vertices A(2, 3), B(4, 7), C(6, 1).

Solution:

- G = ((2+4+6)/3, (3+7+1)/3)
- G = (12/3, 11/3)
- G = (4, 11/3)

9. SPECIAL TYPES OF TRIANGLES

Equilateral Triangle

- All three sides equal
- Use distance formula to verify: AB = BC = CA

Isosceles Triangle

- Two sides equal
- Use distance formula to verify

Right-Angled Triangle

- Pythagoras theorem: $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2$
- Find all three sides using distance formula
- Check if Pythagoras theorem holds

Example: Show that A(3, 0), B(6, 4), C(−1, 3) form a right-angled triangle.

Solution:

- $AB = \sqrt{[(6-3)^2 + (4-0)^2]} = \sqrt{[9+16]} = \sqrt{25} = 5$
- $BC = \sqrt{[(-1-6)^2 + (3-4)^2]} = \sqrt{[49+1]} = \sqrt{50} = 5\sqrt{2}$
- $CA = \sqrt{[(3-(-1))^2 + (0-3)^2]} = \sqrt{[16+9]} = \sqrt{25} = 5$

Check: $BC^2 = AB^2 + CA^2$

- $(5\sqrt{2})^2 = 5^2 + 5^2$
- 50 = 25 + 25
- 50 = 50 ✓
- Triangle is right-angled at A

10. AREA OF QUADRILATERAL

For quadrilateral with vertices A, B, C, D (in order):

Method 1: Divide into two triangles

- Area = Area(△ABC) + Area(△ACD)

Method 2: Using formula (if vertices are in order)

Area = $1/2 |x_1(y_2-y_4) + x_2(y_3-y_1) + x_3(y_4-y_2) + x_4(y_1-y_3)|$

Example: Find the area of quadrilateral with vertices A(1, 2), B(6, 2), C(5, 5), D(2, 4).

Solution (Method 1):

- Area(△ABC) = $1/2 |1(2-5) + 6(5-2) + 5(2-2)| = 1/2 |-3 + 18 + 0| = 1/2 \times 15 = 7.5$
- Area(△ACD) = $1/2 |1(5-4) + 5(4-2) + 2(2-5)| = 1/2 |1 + 10 - 6| = 1/2 \times 5 = 2.5$
- Total Area = 7.5 + 2.5 = 10 square units

11. IMPORTANT PROPERTIES AND RESULTS

Property 1: The coordinates of a point dividing a line segment in a given ratio can be found using section formula.

Property 2: The midpoint divides the line segment in ratio 1:1.

Property 3: Three points are collinear if the area of triangle formed by them is zero.

Property 4: Distance is always positive or zero.

Property 5: If a point lies on x-axis, $y = 0$; if on y-axis, $x = 0$.

Property 6: The centroid divides each median in the ratio 2:1.

Property 7: In a parallelogram, diagonals bisect each other (their midpoints coincide).

Property 8: For a square with vertices A, B, C, D:

- All sides equal: AB = BC = CD = DA
- Diagonals equal: AC = BD
- Diagonals perpendicular (can be checked using slopes in higher classes)

12. FORMULAS AT A GLANCE

S.No.	Formula	Description
1	$\sqrt{[(x_2-x_1)^2 + (y_2-y_1)^2]}$	Distance between two points
2	$((mx_2+nx_1)/(m+n), (my_2+ny_1)/(m+n))$	Section formula (internal)
3	$((x_1+x_2)/2, (y_1+y_2)/2)$	Midpoint formula
4	$1/2\|x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)\|$	Area of triangle
5	$((x_1+x_2+x_3)/3, (y_1+y_2+y_3)/3)$	Centroid of triangle
6	Area = 0	Condition for collinearity

13. PROBLEM-SOLVING STRATEGIES

Step-by-Step Approach:

Step 1: Read the problem carefully and identify what is given and what needs to be found.

Step 2: Plot the points on a rough diagram (helps visualization).

Step 3: Identify which formula to use:

- Finding distance → Distance formula
- Finding dividing point → Section formula
- Finding area → Area formula
- Checking collinearity → Area = 0

Step 4: Substitute values carefully (watch signs!).

Step 5: Simplify and solve.

Step 6: Verify answer if possible.

Common Problem Types:

Type 1: Distance Problems

- Use distance formula
- For finding unknown coordinate, use √(...) = given distance

Type 2: Section Formula Problems

- Internal division → Use section formula
- Finding ratio → Set up equations

Type 3: Area Problems

- Use area formula
- For collinearity, check if area = 0

Type 4: Geometric Figure Verification

- Find all sides using distance formula
- Check properties (equal sides, Pythagoras, etc.)

Type 5: Finding Coordinates

- Use given conditions
- Apply appropriate formulas
- Solve resulting equations

14. WORKED EXAMPLES

Example 1: Find the ratio in which the point P(−3, k) divides the line segment joining A(−5, −4) and B(−2, 3). Also find k.

Solution: Let the ratio be m:1

Using section formula:

- $-3 = (m\times(-2) + 1\times(-5))/(m+1)$
- $-3(m+1) = -2m - 5$
- $-3m - 3 = -2m - 5$
- $-m = -2$
- $m = 2$

So ratio is 2:1

For k:

- $k = (2\times3 + 1\times(-4))/(2+1)$
- $k = (6 - 4)/3$
- $k = 2/3$

Answer: Ratio = 2:1, k = 2/3

Example 2: Find the area of a triangle formed by the points A(5, 2), B(4, 7), C(7, −4).

Solution: Using area formula:

- Area = 1/2 $|5(7-(-4)) + 4(-4-2) + 7(2-7)|$
- = 1/2 $|5(11) + 4(-6) + 7(-5)|$
- = 1/2 $|55 - 24 - 35|$
- = 1/2 $|-4|$
- = 1/2 × 4
- = 2 square units

Example 3: Show that the points A(1, 1), B(2, 3), C(0, −1) are collinear.

Solution: For collinearity, area of triangle = 0

- Area = 1/2 $|1(3-(-1)) + 2(-1-1) + 0(1-3)|$
- = 1/2 $|1(4) + 2(-2) + 0|$
- = 1/2 $|4 - 4|$
- = 0

Since area = 0, points are collinear.

Example 4: Find the value of k for which the points (7, −2), (5, 1), (3, k) are collinear.

Solution: For collinearity:

- $7(1-k) + 5(k-(-2)) + 3(-2-1) = 0$
- $7(1-k) + 5(k+2) + 3(-3) = 0$
- $7 - 7k + 5k + 10 - 9 = 0$
- $8 - 2k = 0$
- $2k = 8$
- $k = 4$

Example 5: The points A(4, 7), B(p, 3), C(7, 3) are the vertices of a right triangle, right-angled at B. Find p.

Solution: For right angle at B: $AB^2 + BC^2 = AC^2$

$AB^2 = (p-4)^2 + (3-7)^2 = (p-4)^2 + 16$ $BC^2 = (7-p)^2 + (3-3)^2 = (7-p)^2$ $AC^2 = (7-4)^2 + (3-7)^2 = 9 + 16 = 25$

Applying Pythagoras:

- $(p-4)^2 + 16 + (7-p)^2 = 25$
- $(p-4)^2 + (7-p)^2 = 9$
- $p^2 - 8p + 16 + 49 - 14p + p^2 = 9$
- $2p^2 - 22p + 65 = 9$
- $2p^2 - 22p + 56 = 0$
- $p^2 - 11p + 28 = 0$
- $(p-4)(p-7) = 0$
- $p = 4$ or $p = 7$

If p = 4, point B coincides with A (not valid) If p = 7, point B coincides with C (not valid)

Re-checking: Actually for right angle at B: $AB^2 + BC^2$ should equal AC^2

Let me recalculate:

- $(p-4)^2 + 16 + (7-p)^2 = 25$
- $p^2 - 8p + 16 + 16 + 49 - 14p + p^2 = 25$
- $2p^2 - 22p + 81 = 25$
- $2p^2 - 22p + 56 = 0$
- $p^2 - 11p + 28 = 0$
- $(p-7)(p-4) = 0$

$p = 4$ or $p = 7$

Example 6: Find the coordinates of points which divide the line segment joining A(2, −3) and B(−4, −6) into three equal parts.

Solution: Let P and Q be the two points that divide AB into three equal parts.

P divides AB in ratio 1:2

- $P = ((1\times(-4) + 2\times2)/(1+2), (1\times(-6) + 2\times(-3))/(1+2))$
- $P = ((-4+4)/3, (-6-6)/3)$
- $P = (0, -4)$

Q divides AB in ratio 2:1

- $Q = ((2\times(-4) + 1\times2)/(2+1), (2\times(-6) + 1\times(-3))/(2+1))$
- $Q = ((-8+2)/3, (-12-3)/3)$
- $Q = (-2, -5)$

Answer: $P = (0, -4)$ and $Q = (-2, -5)$

15. IMPORTANT POINTS TO REMEMBER

✓ Always use absolute value for area (area is always positive)

✓ In section formula, the point closer to B gets larger weight

✓ For collinearity, area of triangle must be exactly zero

✓ Distance is always non-negative

✓ Midpoint is a special case of section formula (1:1 ratio)

✓ When squaring in distance formula, negative signs become positive

✓ Plot points on rough diagram to visualize the problem

✓ Check if answer makes geometric sense

✓ In area formula, maintain cyclic order of vertices

✓ Centroid formula uses sum of all coordinates divided by 3

16. QUICK REVISION POINTS

Before Exam - Must Remember:

☐ Distance Formula: $\sqrt{[(x_2-x_1)^2 + (y_2-y_1)^2]}$

☐ Section Formula: $((mx_2+nx_1)/(m+n), (my_2+ny_1)/(m+n))$

☐ Midpoint: $((x_1+x_2)/2, (y_1+y_2)/2)$

☐ Area of Triangle: $1/2|x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)|$

☐ Collinearity condition: Area = 0

☐ Centroid: $((x_1+x_2+x_3)/3, (y_1+y_2+y_3)/3)$

☐ Signs in quadrants: I(+,+), II(−,+), III(−,−), IV(+,−)

☐ On x-axis: y = 0, On y-axis: x = 0

ALL THE BEST! 📊📐✨

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