StudyByU

PROBABILITY

1. INTRODUCTION TO PROBABILITY

Probability is the branch of mathematics that deals with the likelihood or chance of occurrence of events.

Definition: **Probability is a measure of how likely an event is to occur.**

Range: **Probability of any event always lies between 0 and 1 (inclusive).**

- **$0 \leq P(E) \leq 1$**

2. BASIC CONCEPTS AND TERMINOLOGY

Experiment (or Trial)

An operation which can produce some well-defined outcomes.

Examples:

- **Tossing a coin**
- **Throwing a die**
- **Drawing a card from a deck**
- **Selecting a ball from a bag**

Random Experiment

An experiment whose outcome cannot be predicted with certainty.

Examples:

- **Tossing a coin (can't predict heads or tails)**
- **Rolling a die (can't predict which number)**

Outcome

A possible result of a random experiment.

Examples:

- **When tossing a coin: Head (H) or Tail (T)**
- **When rolling a die: 1, 2, 3, 4, 5, or 6**

Sample Space (S)

The set of all possible outcomes of an experiment.

Notation: S or Ω (omega)

Examples:

Tossing a coin: S = {H, T} n(S) = 2

Rolling a die: S = {1, 2, 3, 4, 5, 6} n(S) = 6

Tossing two coins: S = {HH, HT, TH, TT} n(S) = 4

Rolling two dice: S = {(1,1), (1,2), (1,3), ..., (6,6)} n(S) = 36

Event (E)

A subset of the sample space (a collection of one or more outcomes).

Examples:

When rolling a die:

- Event E_1: Getting an even number = {2, 4, 6}
- Event E_2: Getting a prime number = {2, 3, 5}
- Event E_3: Getting a number greater than 4 = {5, 6}

Types of Events

1. Simple Event (Elementary Event) An event having only one outcome.

Example: Getting 3 when rolling a die = {3}

2. Compound Event An event having more than one outcome.

Example: Getting an even number = {2, 4, 6}

3. Sure Event (Certain Event) An event that will definitely occur. Probability = 1

Example: Getting a number less than 7 when rolling a die

4. Impossible Event An event that cannot occur. Probability = 0

Example: Getting 7 when rolling a die

5. Complementary Event The event "not E" is the complement of event E.

Notation: E' or $\bar{E}$ or $\bar{E}$

Property: P(E) + P(E') = 1 or P(E') = 1 − P(E)

3. CLASSICAL (THEORETICAL) PROBABILITY

Definition

If there are n equally likely outcomes and m of them are favorable to event E, then:

P(E) = Number of favorable outcomes / Total number of possible outcomes

P(E) = m/n = n(E)/n(S)

Where:

- **n(E) = Number of outcomes favorable to E**
- **n(S) = Total number of outcomes in sample space**

Important Properties

Property 1: $0 \leq P(E) \leq 1$

Property 2: P(Sure Event) = 1

Property 3: P(Impossible Event) = 0

Property 4: P(E) + P(not E) = 1 or P(E') = 1 – P(E)

Property 5: If $E_1, E_2, \ldots, E_n$ are all possible outcomes, then: $P(E_1) + P(E_2) + \ldots + P(E_n) = 1$

4. PLAYING CARDS - IMPORTANT FACTS

Standard Deck of Cards

Total cards = 52

Composition:

4 Suits × 13 cards each = 52 cards

Suits:

1. ♠ Spades (Black) - 13 cards

2. ♣ Clubs (Black) - 13 cards

3. ♥ Hearts (Red) - 13 cards

4. ♦ Diamonds (Red) - 13 cards

Each suit has:

- Number cards: 2, 3, 4, 5, 6, 7, 8, 9, 10 (9 cards)

- Face cards: Jack (J), Queen (Q), King (K) (3 cards)

- Ace (A) (1 card)

Total per suit = 13 cards

Important Counts:

- **Red cards = 26 (Hearts + Diamonds)**
- **Black cards = 26 (Spades + Clubs)**
- **Face cards = 12 (4 Jacks + 4 Queens + 4 Kings)**
- **Aces = 4 (one per suit)**
- **Kings = 4**
- **Queens = 4**
- **Jacks = 4**
- **Red Kings = 2**
- **Black Queens = 2**
- **Number cards = 36 (9 per suit × 4 suits)**

5. DICE - IMPORTANT FACTS

Single Die

Total outcomes = 6

Sample Space: {1, 2, 3, 4, 5, 6}

Important subsets:

- **Even numbers: {2, 4, 6} → 3 outcomes**
- **Odd numbers: {1, 3, 5} → 3 outcomes**
- **Prime numbers: {2, 3, 5} → 3 outcomes**
- **Composite numbers: {4, 6} → 2 outcomes**
- **Numbers > 4: {5, 6} → 2 outcomes**
- **Numbers ≤ 2: {1, 2} → 2 outcomes**

Two Dice

Total outcomes = 36

Sample Space (showing as ordered pairs):

(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)

(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)

(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)

(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)

(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)

(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)

Sum of two dice:

Sum	Favorable Outcomes	Count
2	(1,1)	1
3	(1,2), (2,1)	2
4	(1,3), (2,2), (3,1)	3
5	(1,4), (2,3), (3,2), (4,1)	4
6	(1,5), (2,4), (3,3), (4,2), (5,1)	5
7	(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)	6
8	(2,6), (3,5), (4,4), (5,3), (6,2)	5
9	(3,6), (4,5), (5,4), (6,3)	4
10	(4,6), (5,5), (6,4)	3
11	(5,6), (6,5)	2
12	(6,6)	1

Note: Sum of 7 is most likely (6/36 = 1/6)

Doublets (same number on both dice): {(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)} → 6 outcomes

6. COINS - IMPORTANT FACTS

Single Coin

Total outcomes = 2

Sample Space: {H, T}

P(Head) = 1/2 P(Tail) = 1/2

Two Coins

Total outcomes = 4

Sample Space: {HH, HT, TH, TT}

- **Both heads: {HH} → 1 outcome**
- **Both tails: {TT} → 1 outcome**
- **One head, one tail: {HT, TH} → 2 outcomes**
- **At least one head: {HH, HT, TH} → 3 outcomes**
- **At least one tail: {HT, TH, TT} → 3 outcomes**

Three Coins

Total outcomes = 8

Sample Space: {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

- **All heads: {HHH} → 1 outcome**
- **All tails: {TTT} → 1 outcome**
- **Exactly 2 heads: {HHT, HTH, THH} → 3 outcomes**
- **Exactly 2 tails: {HTT, THT, TTH} → 3 outcomes**
- **At least 2 heads: {HHH, HHT, HTH, THH} → 4 outcomes**
- **At most 1 head: {HTT, THT, TTH, TTT} → 4 outcomes**

General Formula

For n coins: Total outcomes = 2^n

Examples:

- **1 coin: $2^1 = 2$**
- **2 coins: $2^2 = 4$**
- **3 coins: $2^3 = 8$**
- **4 coins: $2^4 = 16$**

7. WORKED EXAMPLES

Example 1: A die is thrown once. Find the probability of getting: (i) a prime number (ii) a number greater than 4 (iii) a number less than or equal to 2

Solution: Sample space S = {1, 2, 3, 4, 5, 6} n(S) = 6

(i) Prime number: Prime numbers = {2, 3, 5} $n(E_1) = 3$ P(prime) = 3/6 = 1/2

(ii) Number greater than 4: Numbers > 4 = {5, 6} $n(E_2) = 2$ P(>4) = 2/6 = 1/3

(iii) Number ≤ 2: Numbers ≤ 2 = {1, 2} $n(E_3) = 2$ P(≤2) = 2/6 = 1/3

Example 2: A card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting: (i) a king (ii) a red card (iii) a face card (iv) the queen of diamonds

Solution: Total cards n(S) = 52

(i) A king: Number of kings = 4 P(king) = 4/52 = 1/13

(ii) A red card: Red cards = 26 P(red) = 26/52 = 1/2

(iii) A face card: Face cards = 12 (4 Kings + 4 Queens + 4 Jacks) P(face card) = 12/52 = 3/13

(iv) Queen of diamonds: There is only 1 queen of diamonds P(queen of diamonds) = 1/52 = 1/52

Example 3: Two coins are tossed simultaneously. Find the probability of getting: (i) at least one head (ii) at most one head (iii) no head

Solution: Sample space S = {HH, HT, TH, TT} n(S) = 4

(i) At least one head: E_1 = {HH, HT, TH} $n(E_1) = 3$ P(at least one head) = 3/4 = 3/4

(ii) At most one head: E_2 = {HT, TH, TT} $n(E_2) = 3$ P(at most one head) = 3/4 = 3/4

(iii) No head: E_3 = {TT} $n(E_3) = 1$ P(no head) = 1/4 = 1/4

Example 4: Two dice are thrown together. Find the probability that: (i) the sum of numbers on both dice is 8 (ii) the product is 12 (iii) both dice show the same number

Solution: Total outcomes when two dice are thrown = 36

(i) Sum = 8: Favorable outcomes: {(2,6), (3,5), (4,4), (5,3), (6,2)} $n(E_1) = 5$ P(sum = 8) = 5/36 = 5/36

(ii) Product = 12: Favorable outcomes: {(2,6), (3,4), (4,3), (6,2)} $n(E_2) = 4$ P(product = 12) = 4/36 = 1/9

(iii) Both show same number (doublets): Favorable outcomes: {(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)} $n(E_3) = 6$ P(doublets) = 6/36 = 1/6

Example 5: A bag contains 5 red balls, 8 white balls, and 4 green balls. One ball is drawn at random. Find the probability that it is: (i) red (ii) not green (iii) either white or green

Solution: Total balls = 5 + 8 + 4 = 17

(i) Red ball: P(red) = 5/17 = 5/17

(ii) Not green: Not green = Red + White = 5 + 8 = 13 P(not green) = 13/17 = 13/17

OR using complement: P(not green) = 1 − P(green) = 1 − 4/17 = 13/17

(iii) Either white or green: White or green = 8 + 4 = 12 P(white or green) = 12/17 = 12/17

Example 6: A number is selected at random from numbers 1 to 30. Find the probability that it is: (i) a prime number (ii) a multiple of 3 (iii) a perfect square

Solution: Total numbers = 30

(i) Prime number: Primes from 1 to 30: {2, 3, 5, 7, 11, 13, 17, 19, 23, 29} Count = 10 P(prime) = 10/30 = 1/3

(ii) Multiple of 3: Multiples of 3: {3, 6, 9, 12, 15, 18, 21, 24, 27, 30} Count = 10 P(multiple of 3) = 10/30 = 1/3

(iii) Perfect square: Perfect squares: {1, 4, 9, 16, 25} Count = 5 P(perfect square) = 5/30 = 1/6

Example 7: A box contains cards numbered from 11 to 60. A card is drawn at random. Find the probability that the number on the card is: (i) a perfect square (ii) divisible by 6 (iii) a prime number less than 20

Solution: Total cards = 60 − 11 + 1 = 50

(i) Perfect square: Perfect squares from 11 to 60: {16, 25, 36, 49} Count = 4 P(perfect square) = 4/50 = 2/25

(ii) Divisible by 6: Numbers divisible by 6: {12, 18, 24, 30, 36, 42, 48, 54, 60} Count = 9 P(divisible by 6) = 9/50 = 9/50

(iii) Prime less than 20: Primes less than 20 (from 11 onwards): {11, 13, 17, 19} Count = 4 P(prime < 20) = 4/50 = 2/25

Example 8: Three coins are tossed together. Find the probability of getting: (i) exactly two heads (ii) at least two heads (iii) at most two heads

Solution: Sample space = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT} n(S) = 8

(i) Exactly two heads: E_1 = {HHT, HTH, THH} $n(E_1)$ = 3 P(exactly 2 heads) = 3/8 = 3/8

(ii) At least two heads: E_2 = {HHH, HHT, HTH, THH} $n(E_2)$ = 4 P(at least 2 heads) = 4/8 = 1/2

(iii) At most two heads: E_3 = {HHT, HTH, THH, HTT, THT, TTH, TTT} $n(E_3)$ = 7 P(at most 2 heads) = 7/8 = 7/8

Example 9: If P(E) = 0.35, find P(not E).

Solution: We know: P(E) + P(not E) = 1

P(not E) = 1 – P(E) = 1 – 0.35 = 0.65

Example 10: Two dice are thrown at the same time. Find the probability that the sum of the two numbers appearing on the top is: (i) less than 5 (ii) more than 10

Solution: Total outcomes = 36

(i) Sum < 5: Sum = 2: {(1,1)} → 1 outcome Sum = 3: {(1,2), (2,1)} → 2 outcomes Sum = 4: {(1,3), (2,2), (3,1)} → 3 outcomes

Total favorable = 1 + 2 + 3 = 6 P(sum < 5) = 6/36 = 1/6

(ii) Sum > 10: Sum = 11: {(5,6), (6,5)} → 2 outcomes Sum = 12: {(6,6)} → 1 outcome

Total favorable = 2 + 1 = 3 P(sum > 10) = 3/36 = 1/12

8. COMPLEMENTARY EVENTS

Concept: If E is an event, then "not E" (or E') is the complementary event.

Relationship: P(E) + P(E') = 1

Therefore: P(E') = 1 – P(E)

This is very useful when finding P(not E) is easier than finding P(E).

Example: Instead of finding "probability of getting at least one head in 3 tosses", it's easier to find:

P(at least one head) = 1 – P(no heads) = 1 – P(all tails) = 1 – 1/8 = 7/8

9. MUTUALLY EXCLUSIVE EVENTS

Definition: Two events are mutually exclusive if they cannot occur simultaneously.

Example: When rolling a die:

- Event A: Getting an even number {2, 4, 6}
- Event B: Getting an odd number {1, 3, 5}

These are mutually exclusive (can't get both even and odd simultaneously).

For mutually exclusive events: P(A or B) = P(A) + P(B)

10. IMPORTANT PROBABILITY RESULTS

Result 1: Probability of an impossible event = 0

Result 2: Probability of a sure event = 1

Result 3: $0 \leq P(E) \leq 1$ for any event E

Result 4: $P(E) + P(\bar{E}) = 1$

Result 5: If an event is more likely to happen than not happen: $P(E) > 1/2$

Result 6: If an event is less likely to happen: $P(E) < 1/2$

Result 7: If an event is equally likely to happen or not happen: $P(E) = 1/2$

11. PROBLEM-SOLVING STRATEGIES

Step 1: Understand the Problem

- Read carefully
- Identify what is asked
- Identify the random experiment

Step 2: Find Sample Space

- List all possible outcomes
- Count total outcomes n(S)

Step 3: Find Favorable Outcomes

- List outcomes favorable to event E
- Count favorable outcomes n(E)

Step 4: Apply Formula

- $P(E) = n(E) / n(S)$
- Simplify the fraction
- Check if answer is between 0 and 1

Step 5: Use Complement if Easier

- If finding P(E) is difficult, find P(E')
- Then $P(E) = 1 - P(E')$

12. COMMON PROBLEM TYPES

Type 1: Single Event

- One die, one coin, one card
- Direct application of $P(E) = n(E)/n(S)$

Type 2: Multiple Events

- Two dice, three coins, etc.
- First find total outcomes (product rule)

Type 3: Cards

- Remember: 52 cards, 26 red, 26 black, 12 face cards
- Apply formula directly

Type 4: Balls in Bag

- Count total balls
- Count favorable color/type
- Apply formula

Type 5: Number Selection

- Find total numbers in range
- Count favorable (prime, even, perfect square, etc.)

Type 6: Complementary Events

- Use P(E) = 1 – P(E')
- Useful for "at least" problems

13. KEY TERMS SUMMARY

Term	Meaning
Experiment	An action that produces outcomes
Sample Space (S)	Set of all possible outcomes
Event (E)	Subset of sample space
Favorable Outcomes	Outcomes that satisfy the event
Probability P(E)	n(E) / n(S)
Complementary Event (E')	Event "not E"
Sure Event	P(E) = 1
Impossible Event	P(E) = 0
Equally Likely	All outcomes have same probability

14. IMPORTANT PROBABILITY VALUES (QUICK REFERENCE)

Single Die:

- P(any specific number) = 1/6
- P(even number) = 1/2
- P(odd number) = 1/2
- P(prime number) = 1/2

Single Coin:

- P(Head) = 1/2
- P(Tail) = 1/2

Two Dice:

- P(sum = 7) = 1/6 (most likely)
- P(doublets) = 1/6
- P(sum = 2 or 12) = 1/36 each

Deck of Cards:

- P(any specific card) = 1/52
- P(red card) = 1/2

- P(face card) = 3/13
- P(king) = 1/13
- P(ace) = 1/13

15. FORMULAS AT A GLANCE

Basic Formula: P(E) = Number of favorable outcomes / Total number of outcomes

Complement Rule: $P(E') = 1 - P(E)$

Sum of Probabilities: $P(E_1) + P(E_2) + \ldots + P(E_n) = 1$ (for all possible outcomes)

Range: $0 \leq P(E) \leq 1$

16. TIPS FOR BOARD EXAMS

For 1-2 Mark Questions:

- Write sample space
- Count favorable outcomes
- Apply formula
- Simplify answer

For 3 Mark Questions:

- List complete sample space
- Identify favorable outcomes clearly
- Show calculation step-by-step
- Write answer in simplest form

Time Management:

- 1 mark = 1 minute
- 2 marks = 2 minutes
- 3 marks = 4 minutes

Accuracy Tips:

- Always check if answer is between 0 and 1
- Simplify fractions completely
- Double-check counting of outcomes
- Verify sample space is complete

17. QUICK REVISION CHECKLIST

☐ Definition of probability

☐ Sample space concept

☐ **Difference between outcome and event**

☐ **Formula P(E) = n(E)/n(S)**

☐ **Complementary events: P(E') = 1 – P(E)**

☐ **Properties: 0 ≤ P(E) ≤ 1**

☐ **Playing cards facts (52 cards, suits, face cards)**

☐ **Dice outcomes (single die = 6, two dice = 36)**

☐ **Coin outcomes (n coins = 2^n outcomes)**

☐ **At least, at most, exactly - interpretations**

☐ **All worked examples practiced**

ALL THE BEST! 🎲🃏🎯✨

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