SURFACE AREAS AND VOLUMES

1. INTRODUCTION

Surface Area and Volume deals with measuring the outer area and space occupied by three-dimensional objects (solids).

Key Concepts:

- Surface area of solids
- Volume of solids
- Conversion of one solid into another
- Combination of solids
- Frustum of a cone

2. BASIC DEFINITIONS

3D Solid: An object having length, breadth, and height (three dimensions)

Surface Area: Total area of all outer surfaces of a solid

Types of Surface Area:

- Lateral/Curved Surface Area (CSA): Area of curved/lateral surfaces only
- Total Surface Area (TSA): Area of all surfaces including top and bottom

Volume: Amount of space occupied by a solid (measured in cubic units)

3. CUBOID

Description:

```
     E __________ H
     /|         /|
    / |        / |
 F /__|_____ G  |
   |  |      |  |
   | A|______|__|D
   | /       | /
   |/________|/
   B          C


Length (l), Breadth (b), Height (h)
```

Properties:

- 6 rectangular faces
- 12 edges
- 8 vertices
- Opposite faces are equal and parallel

Formulas:

1. Lateral Surface Area (LSA/CSA): $LSA = 2h(l + b)$ [Area of four vertical faces]

2. Total Surface Area (TSA): $TSA = 2(lb + bh + hl)$ [Area of all 6 faces]

3. Volume: $V = l \times b \times h$

4. Diagonal: $d = \sqrt{l^2 + b^2 + h^2}$

Example 1: Find the TSA and volume of a cuboid with dimensions 8 cm × 6 cm × 5 cm.

Solution: $l = 8$ cm, $b = 6$ cm, $h = 5$ cm

TSA = 2(lb + bh + hl) = 2(8×6 + 6×5 + 5×8) = 2(48 + 30 + 40) = 2(118) = 236 cm^2

Volume = l × b × h = 8 × 6 × 5 = 240 cm^3

4. CUBE

Description:

```
     E __________ H
     /|          /|
    / |         / |
 F /__|_____ G  |
   |  |      |  |
   | A|______|__|D
   | /       | /
   |/________|/
   B          C


All edges = a
```

Properties:

- Special case of cuboid where l = b = h = a
- All faces are squares
- 6 equal square faces
- 12 equal edges
- 8 vertices

Formulas:

1. Lateral Surface Area: LSA = $4a^2$

2. Total Surface Area: TSA = $6a^2$

3. Volume: $V = a^3$

4. Diagonal: $d = a\sqrt{3}$

Example 2: If the edge of a cube is 10 cm, find its TSA and volume.

Solution: a = 10 cm

$TSA = 6a^2 = 6 \times 10^2 = 6 \times 100 = 600\ cm^2$

$Volume = a^3 = 10^3 = 1000\ cm^3$

5. CYLINDER

Description:

```
      _______
     /       \
    |    •    |  ← Top circular face (radius r)
    |         |
    |         |  ← Curved surface (height h)
    |         |
    |    •    |  ← Bottom circular face (radius r)
     \_______/

r = radius, h = height
```

Properties:

- Two circular bases (top and bottom)
- One curved surface
- Circular cross-section throughout

Formulas:

1. Curved Surface Area (CSA): $CSA = 2\pi rh$ [Area of curved portion only]

2. Total Surface Area (TSA): TSA = $2\pi r(r + h)$ OR TSA = $2\pi rh + 2\pi r^2$ [Curved surface + Two circular bases]

3. Volume: $V = \pi r^2 h$

Example 3: Find the CSA, TSA, and volume of a cylinder with radius 7 cm and height 10 cm.

Solution: r = 7 cm, h = 10 cm

CSA = $2\pi rh = 2 \times (22/7) \times 7 \times 10 = 2 \times 22 \times 10 = 440$ cm²

TSA = $2\pi r(r + h) = 2 \times (22/7) \times 7 \times (7 + 10) = 2 \times 22 \times 17 = 748$ cm²

Volume = $\pi r^2 h = (22/7) \times 7^2 \times 10 = (22/7) \times 49 \times 10 = 22 \times 70 = 1540$ cm³

6. CONE

Description:

```
        A (vertex/apex)
       /|\
      / | \
     /  |h \    l = slant height
    /   |   \   h = vertical height
   /    |    \  r = radius of base
  /     •     \
 /______|______\
B       O       C
     (Base: circle with radius r)

l² = r² + h² (Pythagoras theorem)
```

Properties:

- One circular base
- One curved surface tapering to apex
- One vertex (apex)

Formulas:

1. **Slant Height:** $l = \sqrt{r^2 + h^2}$

2. **Curved Surface Area (CSA):** $CSA = \pi rl$

3. **Total Surface Area (TSA):** $TSA = \pi r(l + r)$ OR $TSA = \pi rl + \pi r^2$ [Curved surface + Circular base]

4. **Volume:** $V = (1/3)\pi r^2 h$

Example 4: Find the CSA, TSA, and volume of a cone with radius 7 cm and height 24 cm.

Solution: $r = 7$ cm, $h = 24$ cm

First, find slant height: $l = \sqrt{r^2 + h^2} = \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25$ cm

$CSA = \pi rl = (22/7) \times 7 \times 25 = 22 \times 25 = 550$ cm²

$TSA = \pi r(l + r) = (22/7) \times 7 \times (25 + 7) = 22 \times 32 = 704$ cm²

$\text{Volume} = (1/3)\pi r^2 h = (1/3) \times (22/7) \times 7^2 \times 24 = (1/3) \times 22 \times 7 \times 24 = (1/3) \times 3696 = 1232$ cm³

7. SPHERE

Description:

```
      ___
    /     \
   |   •   |  ← Centre O
    \ ___ /

All points on surface are equidistant
from centre O
Distance = radius (r)
```

Properties:

- Perfectly round 3D object
- All points on surface equidistant from centre

- No edges or vertices
- One continuous curved surface

Formulas:

1. Surface Area: $SA = 4\pi r^2$

2. Volume: $V = (4/3)\pi r^3$

Example 5: Find the surface area and volume of a sphere with radius 7 cm.

Solution: $r = 7$ cm

Surface Area $= 4\pi r^2 = 4 \times (22/7) \times 7^2 = 4 \times (22/7) \times 49 = 4 \times 22 \times 7 = 616$ cm²

Volume $= (4/3)\pi r^3 = (4/3) \times (22/7) \times 7^3 = (4/3) \times (22/7) \times 343 = (4/3) \times 22 \times 49 = (4 \times 22 \times 49)/3 = 4312/3 = 1437.33$ cm³

8. HEMISPHERE

Description:

```
      ___
    /     \
   |   •   |  ← Half of sphere
   |_______|
   (Circular base)

Half sphere = Hemisphere
Has circular flat base + curved surface
```

Properties:

- Half of a sphere
- One circular flat base
- One curved surface

Formulas:

1. **Curved Surface Area (CSA):** $CSA = 2\pi r^2$ [Half of sphere's surface]

2. **Total Surface Area (TSA):** $TSA = 3\pi r^2$ **OR** $TSA = 2\pi r^2 + \pi r^2$ [Curved surface + Circular base]

3. **Volume:** $V = (2/3)\pi r^3$

Example 6: Find the CSA, TSA, and volume of a hemisphere with radius 7 cm.

Solution: $r = 7$ cm

$CSA = 2\pi r^2 = 2 \times (22/7) \times 7^2 = 2 \times (22/7) \times 49 = 2 \times 22 \times 7 = 308\ cm^2$

$TSA = 3\pi r^2 = 3 \times (22/7) \times 49 = 3 \times 22 \times 7 = 462\ cm^2$

$Volume = (2/3)\pi r^3 = (2/3) \times (22/7) \times 343 = (2/3) \times 22 \times 49 = (2 \times 22 \times 49)/3 = 2156/3 = 718.67\ cm^3$

9. FRUSTUM OF A CONE

Description:

```
     ___________  ← Top (radius r₁)
    /           \
   /             \
  /               \  h = height
 /                 \
/___________________\ ← Bottom (radius r₂)

Frustum = Cone with top cut off parallel to base
r₁ = radius of top (smaller)
r₂ = radius of bottom (larger)
h = height
l = slant height
```

Properties:

- Cone with top portion cut off
- Two circular bases of different radii
- One curved surface

Formulas:

1. **Slant Height:** $l = \sqrt{[h^2 + (r_2 - r_1)^2]}$

2. **Curved Surface Area (CSA):** $CSA = \pi(r_1 + r_2)l$

3. **Total Surface Area (TSA):** $TSA = \pi(r_1 + r_2)l + \pi r_1^2 + \pi r_2^2$ **OR** $TSA = \pi[(r_1 + r_2)l + r_1^2 + r_2^2]$

4. **Volume:** $V = (1/3)\pi h(r_1^2 + r_2^2 + r_1r_2)$

Example 7: A frustum of a cone has radii 3 cm and 10 cm, and height 12 cm. Find its volume and curved surface area.

Solution: $r_1 = 3$ cm, $r_2 = 10$ cm, $h = 12$ cm

Slant height: $l = \sqrt{[h^2 + (r_2 - r_1)^2]} = \sqrt{[12^2 + (10 - 3)^2]} = \sqrt{[144 + 49]} = \sqrt{193} = 13.89$ cm

Volume $= (1/3)\pi h(r_1^2 + r_2^2 + r_1r_2) = (1/3) \times (22/7) \times 12 \times (3^2 + 10^2 + 3\times10) = (1/3) \times (22/7) \times 12 \times (9 + 100 + 30) = (1/3) \times (22/7) \times 12 \times 139 = (22 \times 12 \times 139)/(3 \times 7) = 36696/21 = 1747.43$ cm³

$CSA = \pi(r_1 + r_2)l = (22/7) \times (3 + 10) \times 13.89 = (22/7) \times 13 \times 13.89 = 568.76$ cm²

10. COMBINATION OF SOLIDS

Combination of solids involves finding surface area and volume of objects made by joining two or more basic solids.

Common Combinations:

1. Cylinder + Cone (pencil shape, rocket)
2. Cylinder + Hemisphere (capsule, water tank)
3. Cone + Hemisphere (ice cream cone)
4. Cuboid + Pyramid
5. Two cones joined at base (double cone)

Strategy for Combinations:

For Surface Area:

1. Identify individual shapes
2. Calculate surface area of each part
3. Subtract areas where shapes join (common base)
4. Add remaining surfaces

For Volume:

1. Calculate volume of each solid separately
2. Add all volumes

Example 8: A toy is in the form of a cone mounted on a hemisphere. The radius of the base of the cone is 3.5 cm and its height is 12 cm. Find the surface area and volume of the toy.

Solution:

```
     /\
    /  \
   /    \  ← Cone (h = 12 cm)
  /______\
 |        |
 |   •    | ← Hemisphere (r = 3.5 cm)
  \______/
```

$r = 3.5$ cm, $h = 12$ cm

For Cone: Slant height $l = \sqrt{r^2 + h^2} = \sqrt{3.5^2 + 12^2} = \sqrt{12.25 + 144} = \sqrt{156.25} = 12.5$ cm

CSA of cone $= \pi rl = (22/7) \times 3.5 \times 12.5 = 22 \times 0.5 \times 12.5 = 137.5\ cm^2$

For Hemisphere: CSA of hemisphere $= 2\pi r^2 = 2 \times (22/7) \times 3.5^2 = 2 \times (22/7) \times 12.25 = 2 \times 22 \times 1.75 = 77\ cm^2$

Total Surface Area $= 137.5 + 77 = 214.5\ cm^2$

For Volume: Volume of cone $= (1/3)\pi r^2 h = (1/3) \times (22/7) \times 3.5^2 \times 12 = (1/3) \times (22/7) \times 12.25 \times 12 = 154\ cm^3$

Volume of hemisphere $= (2/3)\pi r^3 = (2/3) \times (22/7) \times 3.5^3 = (2/3) \times (22/7) \times 42.875 = 89.83\ cm^3$

Total Volume $= 154 + 89.83 = 243.83\ cm^3$

11. CONVERSION OF SOLIDS

When one solid is melted and recast into another shape, volume remains constant.

Principle: Volume of original solid = Volume of new solid

Example 9: A solid sphere of radius 6 cm is melted and recast into small spherical balls each of radius 0.3 cm. Find the number of balls formed.

Solution: Radius of large sphere (R) = 6 cm Radius of small ball (r) = 0.3 cm

Volume of large sphere = $(4/3)\pi R^3 = (4/3)\pi \times 6^3 = (4/3)\pi \times 216 = 288\pi$ cm³

Volume of one small ball = $(4/3)\pi r^3 = (4/3)\pi \times (0.3)^3 = (4/3)\pi \times 0.027 = 0.036\pi$ cm³

Number of balls = Volume of large sphere / Volume of one ball = $288\pi / 0.036\pi = 288 / 0.036$ = 8000 balls

Example 10: A cylindrical vessel of radius 8 cm contains water. A solid sphere of radius 6 cm is dropped into it. Find the rise in water level.

Solution:

```
Before:                 After:

┌─────┐                 ┌─────┐
│     │                 │  ○  │  ← Sphere
│     │                 │     │
│~~~~~│  ← Water        │~~~~~│  ← Water rises by h
└─────┘                 └─────┘
```

Radius of cylinder (r) = 8 cm Radius of sphere (R) = 6 cm

Volume of sphere = $(4/3)\pi R^3 = (4/3) \times \pi \times 6^3 = (4/3) \times \pi \times 216 = 288\pi$ cm³

This volume equals volume of water displaced = $\pi r^2 h$

$\pi r^2 h = 288\pi$ $\pi \times 8^2 \times h = 288\pi$ $64h = 288$ $h = 288/64$ $h = 4.5$ cm

12. IMPORTANT FORMULAS - QUICK REFERENCE

Solid	CSA/LSA	TSA	Volume
Cuboid	2h(l+b)	2(lb+bh+hl)	lbh
Cube	$4a^2$	$6a^2$	a^3
Cylinder	$2\pi rh$	$2\pi r(r+h)$	πr^2h
Cone	πrl	$\pi r(l+r)$	$(1/3)\pi r^2h$
Sphere	$4\pi r^2$	$4\pi r^2$	$(4/3)\pi r^3$
Hemisphere	$2\pi r^2$	$3\pi r^2$	$(2/3)\pi r^3$
Frustum	$\pi(r_1+r_2)l$	$\pi[(r_1+r_2)l+r_1^2+r_2^2]$	$(1/3)\pi h(r_1^2+r_2^2+r_1r_2)$

13. WORKED EXAMPLES

Example 11: A hemispherical bowl of internal radius 9 cm is full of water. This water is to be filled into cylindrical bottles of diameter 3 cm and height 4 cm. Find the number of bottles required.

Solution: Radius of hemisphere = 9 cm Radius of cylinder = 3/2 = 1.5 cm Height of cylinder = 4 cm

Volume of hemisphere = $(2/3)\pi r^3 = (2/3) \times \pi \times 9^3 = (2/3) \times \pi \times 729 = 486\pi$ cm³

Volume of one bottle = $\pi r^2h = \pi \times (1.5)^2 \times 4 = \pi \times 2.25 \times 4 = 9\pi$ cm³

Number of bottles = $486\pi / 9\pi = 54$ bottles

Example 12: A circus tent is cylindrical up to a height of 3 m and conical above it. If the diameter of the base is 105 m and the slant height of the conical part is 53 m, find the total canvas used.

Solution:

```
   /\
  /  \  ← Cone (l = 53 m)
 /____\
 |    |
 |    | ← Cylinder (h = 3 m)
 |____|

d = 105 m, r = 52.5 m
```

d = 105 m, r = 52.5 m

Radius = 105/2 = 52.5 m Height of cylinder = 3 m Slant height of cone = 53 m

CSA of cylindrical part = $2\pi rh = 2 \times (22/7) \times 52.5 \times 3 = 2 \times 22 \times 7.5 \times 3 = 990$ m²

CSA of conical part = $\pi rl = (22/7) \times 52.5 \times 53 = 22 \times 7.5 \times 53 = 8745$ m²

Total canvas = 990 + 8745 = 9735 m²

Example 13: A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

Solution: Radius of sphere (R) = 4.2 cm Radius of cylinder (r) = 6 cm

Volume of sphere = Volume of cylinder $(4/3)\pi R^3 = \pi r^2 h$ $(4/3) \times (4.2)^3 = 6^2 \times h$ $(4/3) \times 74.088 = 36h$ $98.784 = 36h$ $h = 98.784/36$ $h = 2.744$ cm

Example 14: A conical vessel of radius 6 cm and height 8 cm is completely filled with water. A sphere is lowered into the water and its size is such that when it touches the sides, it is just immersed. What fraction of water overflows?

Solution: For a sphere fitting exactly in cone: Radius of sphere = $(r \times h)/(r + l)$

First find slant height: $l = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$ cm

Radius of sphere: $R = (6 \times 8)/(6 + 10) = 48/16 = 3$ cm

Volume of cone = $(1/3)\pi r^2 h = (1/3) \times \pi \times 6^2 \times 8 = (1/3) \times \pi \times 36 \times 8 = 96\pi$ cm³

Volume of sphere = $(4/3)\pi R^3 = (4/3) \times \pi \times 3^3 = (4/3) \times \pi \times 27 = 36\pi$ cm³

Fraction overflowed = $36\pi/96\pi = 36/96 = 3/8$

14. PROBLEM-SOLVING STRATEGIES

Strategy 1: Identify the Solid

- Read carefully and visualize
- Draw a diagram
- Label all dimensions

Strategy 2: Choose Correct Formula

- CSA or TSA? (Check what's asked)
- For combinations, identify each part
- For conversions, equate volumes

Strategy 3: Unit Consistency

- Convert all to same unit (cm, m, etc.)
- Volume in cm^3 or m^3
- Surface area in cm^2 or m^2

Strategy 4: For Combinations

- Surface Area: Add visible surfaces, subtract hidden
- Volume: Simply add all volumes

Strategy 5: For Conversions

- $Volume_1 = Volume_2$
- Find unknown dimension

15. IMPORTANT POINTS TO REMEMBER

✓ For cone: Always find l first if not given

✓ Sphere has only one formula for SA ($4\pi r^2$)

✓ Hemisphere TSA = $3\pi r^2$ (not $2\pi r^2$)

✓ In conversions: $Volume_1 = Volume_2$

✓ In combinations: Subtract common areas

✓ Frustum: $r_2 > r_1$ (larger base at bottom)

✓ Cylinder: TSA = $2\pi r(r + h)$ or $2\pi rh + 2\pi r^2$

✓ Cone volume = (1/3) × cylinder volume (same r, h)

✓ 1 litre = 1000 cm^3

✓ 1 m^3 = 1000000 cm^3

16. QUICK REVISION CHECKLIST

☐ All formulas for 7 solids memorized

☐ Difference between CSA and TSA clear

☐ Slant height formula for cone

☐ Frustum formulas mastered

☐ Combination strategy understood

☐ Conversion principle clear (equal volumes)

☐ Water displacement problems practiced

☐ Cost calculation method known

☐ Unit conversions mastered

☐ All worked examples solved

ALL THE BEST! 📦🔺⚫✨

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