

SURFACE AREAS AND VOLUMES

1. INTRODUCTION

Surface Area and Volume deals with measuring the outer area and space occupied by three-dimensional objects (solids).

Key Concepts:

- Surface area of solids
- Volume of solids
- Conversion of one solid into another
- Combination of solids
- Frustum of a cone

2. BASIC DEFINITIONS

3D Solid: An object having length, breadth, and height (three dimensions)

Surface Area: Total area of all outer surfaces of a solid

Types of Surface Area:

- **Lateral/Curved Surface Area (CSA):** Area of curved/lateral surfaces only
- **Total Surface Area (TSA):** Area of all surfaces including top and bottom

Volume: Amount of space occupied by a solid (measured in cubic units)

3. CUBOID

Description:



Length (l), Breadth (b), Height (h)

Properties:

- 6 rectangular faces
- 12 edges
- 8 vertices
- Opposite faces are equal and parallel

Formulas:

1. Lateral Surface Area (LSA/CSA): $LSA = 2h(l + b)$ [Area of four vertical faces]

2. Total Surface Area (TSA): $TSA = 2(lb + bh + hl)$ [Area of all 6 faces]

3. Volume: $V = l \times b \times h$

4. Diagonal: $d = \sqrt{l^2 + b^2 + h^2}$

Example 1: Find the TSA and volume of a cuboid with dimensions 8 cm \times 6 cm \times 5 cm.

Solution: $l = 8$ cm, $b = 6$ cm, $h = 5$ cm

$$TSA = 2(lb + bh + hl) = 2(8 \times 6 + 6 \times 5 + 5 \times 8) = 2(48 + 30 + 40) = 2(118) = 236 \text{ cm}^2$$

$$\text{Volume} = l \times b \times h = 8 \times 6 \times 5 = 240 \text{ cm}^3$$

4. CUBE

Description:



All edges = a

Properties:

- Special case of cuboid where $l = b = h = a$
- All faces are squares
- 6 equal square faces
- 12 equal edges
- 8 vertices

Formulas:

1. Lateral Surface Area: $LSA = 4a^2$

2. Total Surface Area: $TSA = 6a^2$

3. Volume: $V = a^3$

4. Diagonal: $d = a\sqrt{3}$

Example 2: If the edge of a cube is 10 cm, find its TSA and volume.

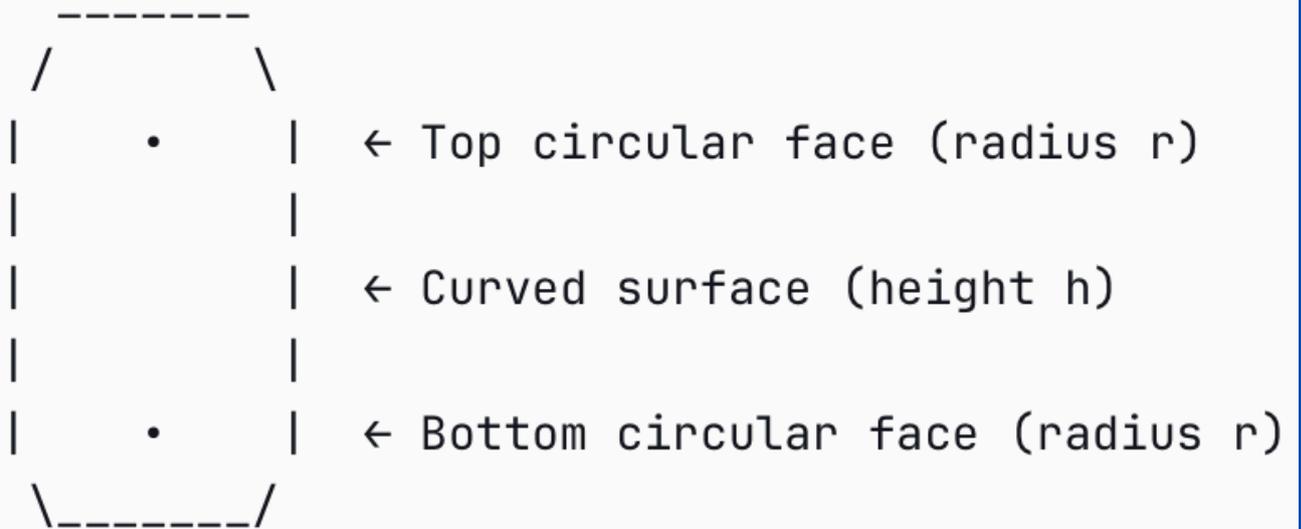
Solution: $a = 10$ cm

$TSA = 6a^2 = 6 \times 10^2 = 6 \times 100 = 600$ cm²

$Volume = a^3 = 10^3 = 1000$ cm³

5. CYLINDER

Description:



$r =$ radius, $h =$ height

Properties:

- Two circular bases (top and bottom)
- One curved surface
- Circular cross-section throughout

Formulas:

1. Curved Surface Area (CSA): $CSA = 2\pi rh$ [Area of curved portion only]

2. Total Surface Area (TSA): $TSA = 2\pi r(r + h)$ OR $TSA = 2\pi rh + 2\pi r^2$ [Curved surface + Two circular bases]

3. Volume: $V = \pi r^2 h$

Example 3: Find the CSA, TSA, and volume of a cylinder with radius 7 cm and height 10 cm.

Solution: $r = 7$ cm, $h = 10$ cm

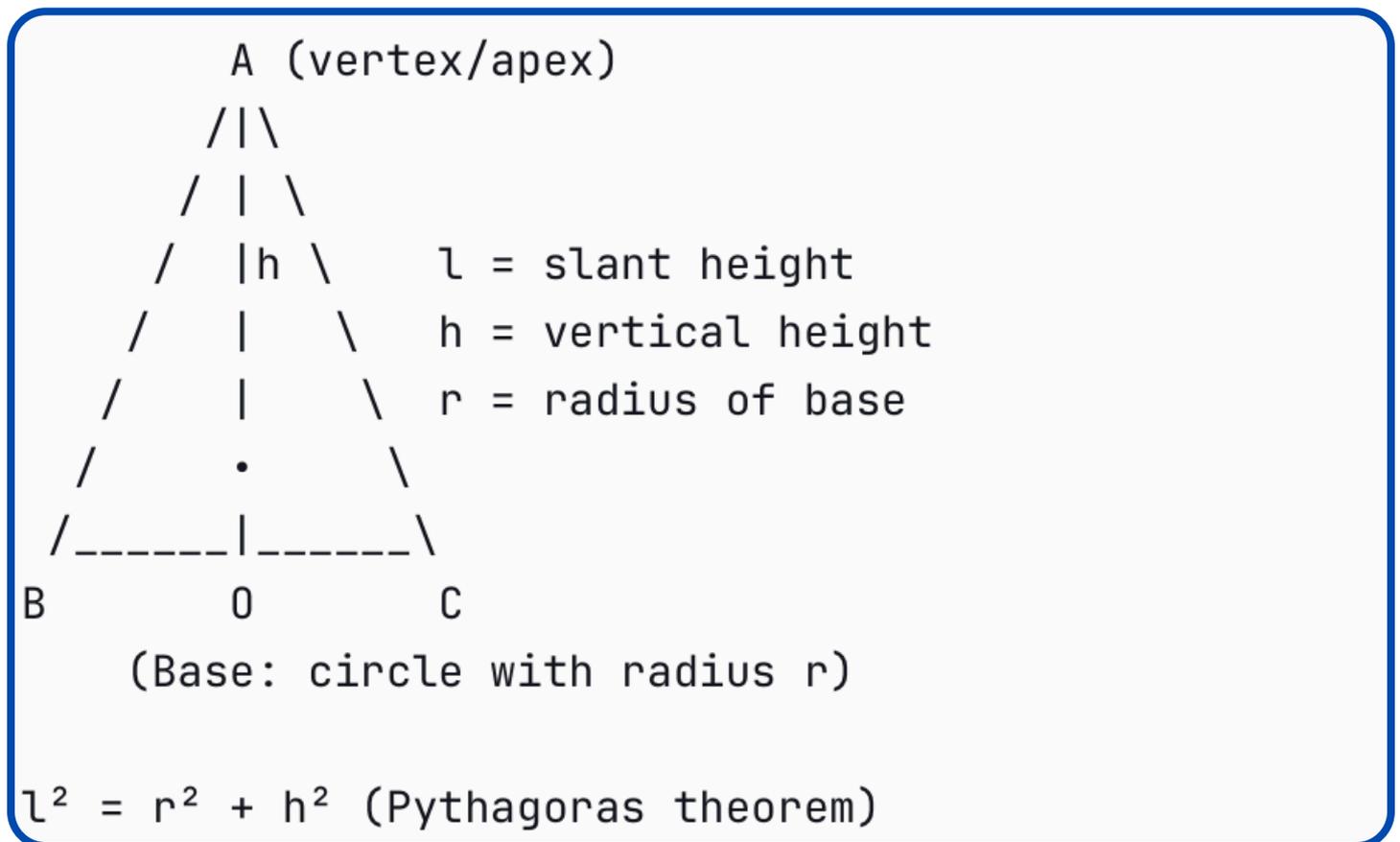
$$CSA = 2\pi rh = 2 \times (22/7) \times 7 \times 10 = 2 \times 22 \times 10 = 440 \text{ cm}^2$$

$$TSA = 2\pi r(r + h) = 2 \times (22/7) \times 7 \times (7 + 10) = 2 \times 22 \times 17 = 748 \text{ cm}^2$$

$$\text{Volume} = \pi r^2 h = (22/7) \times 7^2 \times 10 = (22/7) \times 49 \times 10 = 22 \times 70 = 1540 \text{ cm}^3$$

6. CONE

Description:



Properties:

- One circular base
- One curved surface tapering to apex
- One vertex (apex)

Formulas:

1. Slant Height: $l = \sqrt{r^2 + h^2}$

2. Curved Surface Area (CSA): $CSA = \pi r l$

3. Total Surface Area (TSA): $TSA = \pi r(l + r)$ OR $TSA = \pi r l + \pi r^2$ [Curved surface + Circular base]

4. Volume: $V = (1/3)\pi r^2 h$

Example 4: Find the CSA, TSA, and volume of a cone with radius 7 cm and height 24 cm.

Solution: $r = 7$ cm, $h = 24$ cm

First, find slant height: $l = \sqrt{r^2 + h^2} = \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25$ cm

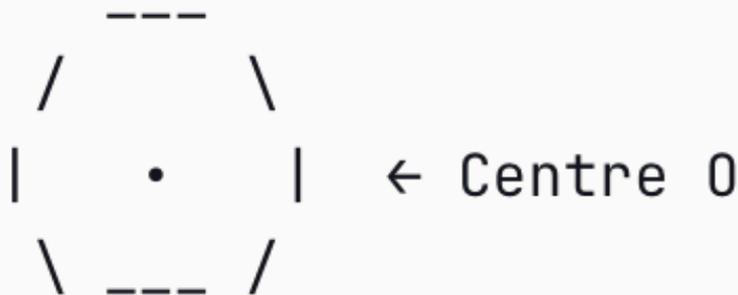
$$CSA = \pi r l = (22/7) \times 7 \times 25 = 22 \times 25 = 550 \text{ cm}^2$$

$$TSA = \pi r(l + r) = (22/7) \times 7 \times (25 + 7) = 22 \times 32 = 704 \text{ cm}^2$$

$$\text{Volume} = (1/3)\pi r^2 h = (1/3) \times (22/7) \times 7^2 \times 24 = (1/3) \times 22 \times 7 \times 24 = (1/3) \times 3696 = 1232 \text{ cm}^3$$

7. SPHERE

Description:



All points on surface are equidistant from centre O

Distance = radius (r)

Properties:

- Perfectly round 3D object
- All points on surface equidistant from centre

- No edges or vertices
- One continuous curved surface

Formulas:

1. Surface Area: $SA = 4\pi r^2$

2. Volume: $V = (4/3)\pi r^3$

Example 5: Find the surface area and volume of a sphere with radius 7 cm.

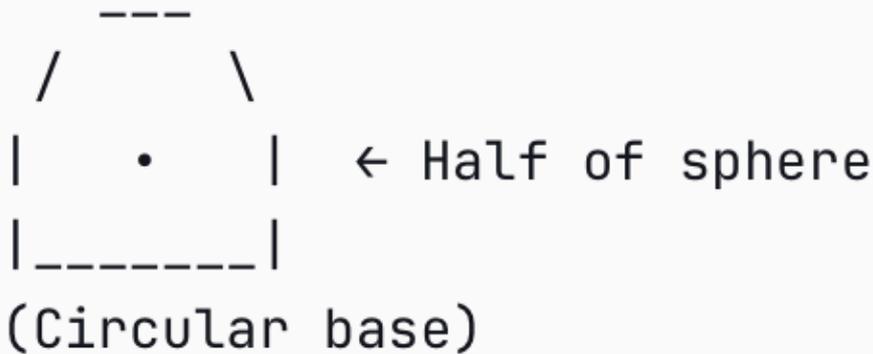
Solution: $r = 7$ cm

Surface Area = $4\pi r^2 = 4 \times (22/7) \times 7^2 = 4 \times (22/7) \times 49 = 4 \times 22 \times 7 = 616 \text{ cm}^2$

Volume = $(4/3)\pi r^3 = (4/3) \times (22/7) \times 7^3 = (4/3) \times (22/7) \times 343 = (4/3) \times 22 \times 49 = (4 \times 22 \times 49)/3 = 4312/3 = 1437.33 \text{ cm}^3$

8. HEMISPHERE

Description:



Half sphere = Hemisphere

Has circular flat base + curved surface

Properties:

- Half of a sphere
- One circular flat base
- One curved surface

Formulas:

1. Curved Surface Area (CSA): $CSA = 2\pi r^2$ [Half of sphere's surface]

2. Total Surface Area (TSA): $TSA = 3\pi r^2$ OR $TSA = 2\pi r^2 + \pi r^2$ [Curved surface + Circular base]

3. Volume: $V = (2/3)\pi r^3$

Example 6: Find the CSA, TSA, and volume of a hemisphere with radius 7 cm.

Solution: $r = 7$ cm

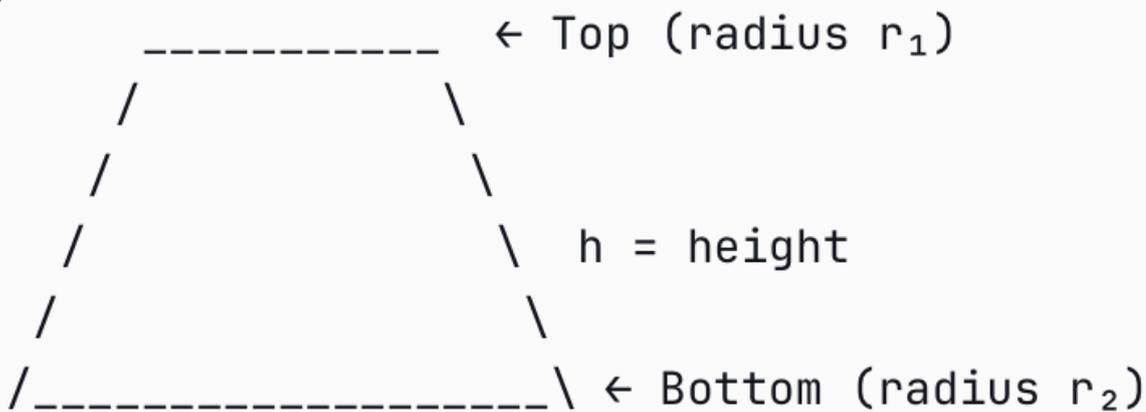
$$CSA = 2\pi r^2 = 2 \times (22/7) \times 7^2 = 2 \times (22/7) \times 49 = 2 \times 22 \times 7 = 308 \text{ cm}^2$$

$$TSA = 3\pi r^2 = 3 \times (22/7) \times 49 = 3 \times 22 \times 7 = 462 \text{ cm}^2$$

$$\text{Volume} = (2/3)\pi r^3 = (2/3) \times (22/7) \times 343 = (2/3) \times 22 \times 49 = (2 \times 22 \times 49)/3 = 2156/3 = 718.67 \text{ cm}^3$$

9. FRUSTUM OF A CONE

Description:



Frustum = Cone with top cut off parallel to base

r_1 = radius of top (smaller)

r_2 = radius of bottom (larger)

h = height

l = slant height

Properties:

- Cone with top portion cut off
- Two circular bases of different radii
- One curved surface

Formulas:

1. Slant Height: $l = \sqrt{h^2 + (r_2 - r_1)^2}$

2. Curved Surface Area (CSA): $CSA = \pi(r_1 + r_2)l$

3. Total Surface Area (TSA): $TSA = \pi(r_1 + r_2)l + \pi r_1^2 + \pi r_2^2$ OR $TSA = \pi[(r_1 + r_2)l + r_1^2 + r_2^2]$

4. Volume: $V = (1/3)\pi h(r_1^2 + r_2^2 + r_1 r_2)$

Example 7: A frustum of a cone has radii 3 cm and 10 cm, and height 12 cm. Find its volume and curved surface area.

Solution: $r_1 = 3$ cm, $r_2 = 10$ cm, $h = 12$ cm

Slant height: $l = \sqrt{h^2 + (r_2 - r_1)^2} = \sqrt{12^2 + (10 - 3)^2} = \sqrt{144 + 49} = \sqrt{193} = 13.89$ cm

Volume = $(1/3)\pi h(r_1^2 + r_2^2 + r_1 r_2) = (1/3) \times (22/7) \times 12 \times (3^2 + 10^2 + 3 \times 10) = (1/3) \times (22/7) \times 12 \times (9 + 100 + 30) = (1/3) \times (22/7) \times 12 \times 139 = (22 \times 12 \times 139)/(3 \times 7) = 36696/21 = 1747.43$ cm³

CSA = $\pi(r_1 + r_2)l = (22/7) \times (3 + 10) \times 13.89 = (22/7) \times 13 \times 13.89 = 568.76$ cm²

10. COMBINATION OF SOLIDS

Combination of solids involves finding surface area and volume of objects made by joining two or more basic solids.

Common Combinations:

1. Cylinder + Cone (pencil shape, rocket)
2. Cylinder + Hemisphere (capsule, water tank)
3. Cone + Hemisphere (ice cream cone)
4. Cuboid + Pyramid
5. Two cones joined at base (double cone)

Strategy for Combinations:

For Surface Area:

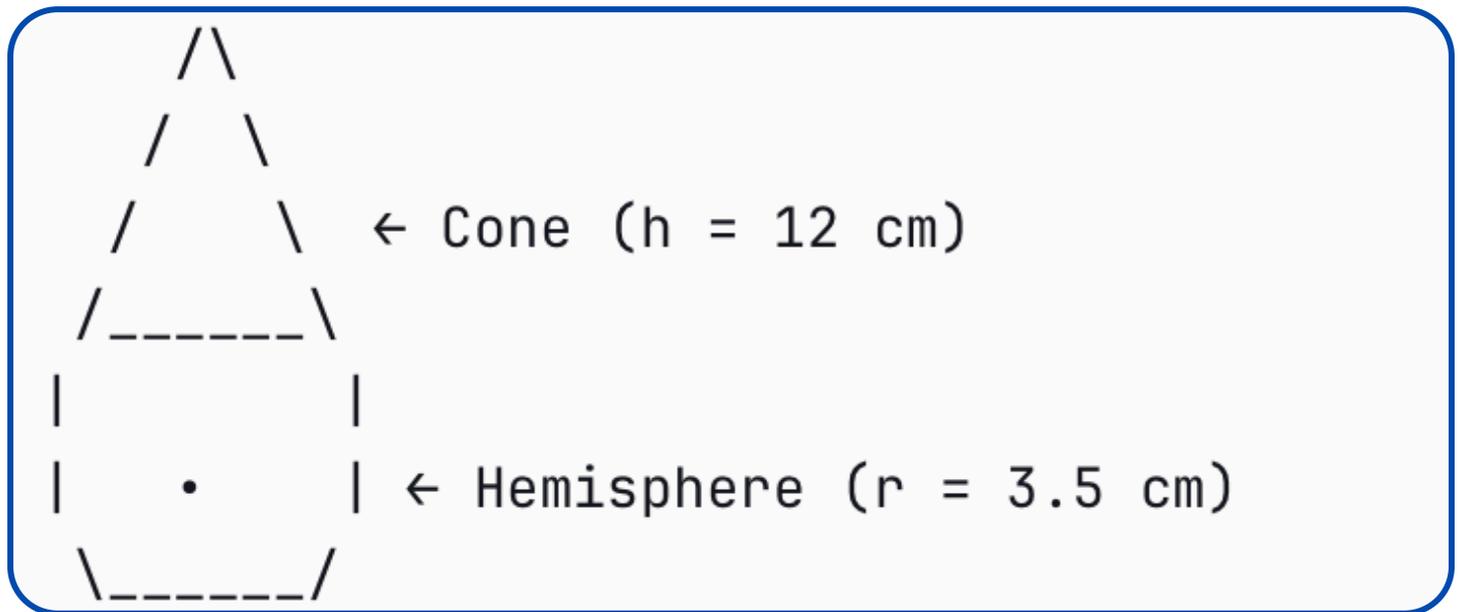
1. Identify individual shapes
2. Calculate surface area of each part
3. Subtract areas where shapes join (common base)
4. Add remaining surfaces

For Volume:

1. Calculate volume of each solid separately
2. Add all volumes

Example 8: A toy is in the form of a cone mounted on a hemisphere. The radius of the base of the cone is 3.5 cm and its height is 12 cm. Find the surface area and volume of the toy.

Solution:



$$r = 3.5 \text{ cm, } h = 12 \text{ cm}$$

$$\text{For Cone: Slant height } l = \sqrt{(r^2 + h^2)} = \sqrt{(3.5^2 + 12^2)} = \sqrt{(12.25 + 144)} = \sqrt{156.25} = 12.5 \text{ cm}$$

$$\text{CSA of cone} = \pi r l = (22/7) \times 3.5 \times 12.5 = 22 \times 0.5 \times 12.5 = 137.5 \text{ cm}^2$$

$$\text{For Hemisphere: CSA of hemisphere} = 2\pi r^2 = 2 \times (22/7) \times 3.5^2 = 2 \times (22/7) \times 12.25 = 2 \times 22 \times 1.75 = 77 \text{ cm}^2$$

$$\text{Total Surface Area} = 137.5 + 77 = 214.5 \text{ cm}^2$$

$$\text{For Volume: Volume of cone} = (1/3)\pi r^2 h = (1/3) \times (22/7) \times 3.5^2 \times 12 = (1/3) \times (22/7) \times 12.25 \times 12 = 154 \text{ cm}^3$$

$$\text{Volume of hemisphere} = (2/3)\pi r^3 = (2/3) \times (22/7) \times 3.5^3 = (2/3) \times (22/7) \times 42.875 = 89.83 \text{ cm}^3$$

$$\text{Total Volume} = 154 + 89.83 = 243.83 \text{ cm}^3$$

11. CONVERSION OF SOLIDS

When one solid is melted and recast into another shape, volume remains constant.

Principle: Volume of original solid = Volume of new solid

Example 9: A solid sphere of radius 6 cm is melted and recast into small spherical balls each of radius 0.3 cm. Find the number of balls formed.

Solution: Radius of large sphere (R) = 6 cm Radius of small ball (r) = 0.3 cm

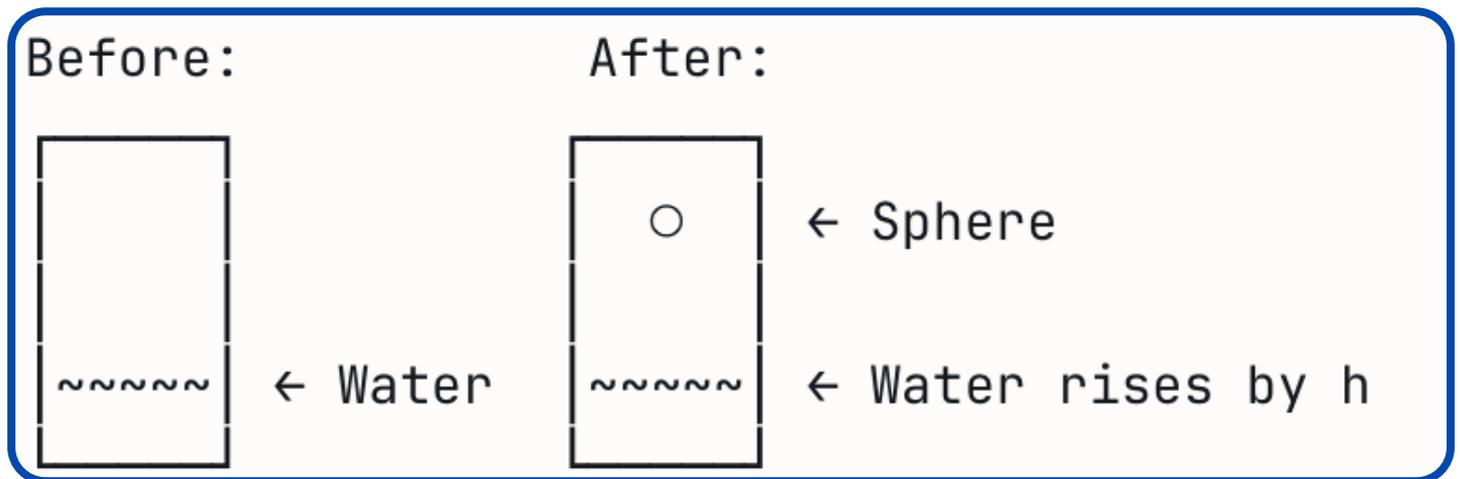
Volume of large sphere = $(4/3)\pi R^3 = (4/3)\pi \times 6^3 = (4/3)\pi \times 216 = 288\pi \text{ cm}^3$

Volume of one small ball = $(4/3)\pi r^3 = (4/3)\pi \times (0.3)^3 = (4/3)\pi \times 0.027 = 0.036\pi \text{ cm}^3$

Number of balls = Volume of large sphere / Volume of one ball = $288\pi / 0.036\pi = 288 / 0.036 = 8000$ balls

Example 10: A cylindrical vessel of radius 8 cm contains water. A solid sphere of radius 6 cm is dropped into it. Find the rise in water level.

Solution:



Radius of cylinder (r) = 8 cm Radius of sphere (R) = 6 cm

Volume of sphere = $(4/3)\pi R^3 = (4/3) \times \pi \times 6^3 = (4/3) \times \pi \times 216 = 288\pi \text{ cm}^3$

This volume equals volume of water displaced = $\pi r^2 h$

$\pi r^2 h = 288\pi \pi \times 8^2 \times h = 288\pi 64h = 288 h = 288/64 h = 4.5 \text{ cm}$

12. IMPORTANT FORMULAS - QUICK REFERENCE

Solid	CSA/LSA	TSA	Volume
Cuboid	$2h(l+b)$	$2(lb+bh+hl)$	lbh
Cube	$4a^2$	$6a^2$	a^3
Cylinder	$2\pi rh$	$2\pi r(r+h)$	$\pi r^2 h$
Cone	πrl	$\pi r(l+r)$	$(1/3)\pi r^2 h$
Sphere	$4\pi r^2$	$4\pi r^2$	$(4/3)\pi r^3$
Hemisphere	$2\pi r^2$	$3\pi r^2$	$(2/3)\pi r^3$
Frustum	$\pi(r_1+r_2)l$	$\pi[(r_1+r_2)l+r_1^2+r_2^2]$	$(1/3)\pi h(r_1^2+r_2^2+r_1r_2)$

13. WORKED EXAMPLES

Example 11: A hemispherical bowl of internal radius 9 cm is full of water. This water is to be filled into cylindrical bottles of diameter 3 cm and height 4 cm. Find the number of bottles required.

Solution: Radius of hemisphere = 9 cm Radius of cylinder = $3/2 = 1.5$ cm Height of cylinder = 4 cm

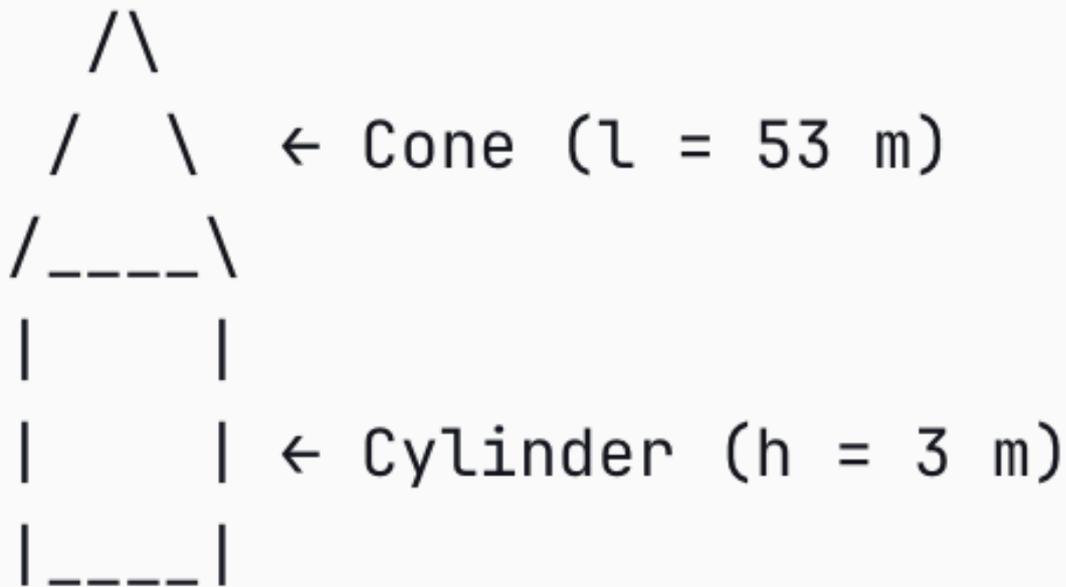
Volume of hemisphere = $(2/3)\pi r^3 = (2/3) \times \pi \times 9^3 = (2/3) \times \pi \times 729 = 486\pi \text{ cm}^3$

Volume of one bottle = $\pi r^2 h = \pi \times (1.5)^2 \times 4 = \pi \times 2.25 \times 4 = 9\pi \text{ cm}^3$

Number of bottles = $486\pi / 9\pi = 54$ bottles

Example 12: A circus tent is cylindrical up to a height of 3 m and conical above it. If the diameter of the base is 105 m and the slant height of the conical part is 53 m, find the total canvas used.

Solution:



$$d = 105 \text{ m}, r = 52.5 \text{ m}$$

$$d = 105 \text{ m}, r = 52.5 \text{ m}$$

Radius = $105/2 = 52.5 \text{ m}$ Height of cylinder = 3 m Slant height of cone = 53 m

CSA of cylindrical part = $2\pi rh = 2 \times (22/7) \times 52.5 \times 3 = 2 \times 22 \times 7.5 \times 3 = 990 \text{ m}^2$

CSA of conical part = $\pi rl = (22/7) \times 52.5 \times 53 = 22 \times 7.5 \times 53 = 8745 \text{ m}^2$

Total canvas = $990 + 8745 = 9735 \text{ m}^2$

Example 13: A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm . Find the height of the cylinder.

Solution: Radius of sphere (R) = 4.2 cm Radius of cylinder (r) = 6 cm

Volume of sphere = Volume of cylinder $(4/3)\pi R^3 = \pi r^2 h$ $(4/3) \times (4.2)^3 = 6^2 \times h$ $(4/3) \times 74.088 = 36h$ $98.784 = 36h$ $h = 98.784/36$ $h = 2.744 \text{ cm}$

Example 14: A conical vessel of radius 6 cm and height 8 cm is completely filled with water. A sphere is lowered into the water and its size is such that when it touches the sides, it is just immersed. What fraction of water overflows?

Solution: For a sphere fitting exactly in cone: Radius of sphere = $(r \times h)/(r + l)$

First find slant height: $l = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10 \text{ cm}$

Radius of sphere: $R = (6 \times 8)/(6 + 10) = 48/16 = 3 \text{ cm}$

Volume of cone = $(1/3)\pi r^2 h = (1/3) \times \pi \times 6^2 \times 8 = (1/3) \times \pi \times 36 \times 8 = 96\pi \text{ cm}^3$

Volume of sphere = $(4/3)\pi R^3 = (4/3) \times \pi \times 3^3 = (4/3) \times \pi \times 27 = 36\pi \text{ cm}^3$

Fraction overflowed = $36\pi/96\pi = 36/96 = 3/8$

14. PROBLEM-SOLVING STRATEGIES

Strategy 1: Identify the Solid

- Read carefully and visualize
- Draw a diagram
- Label all dimensions

Strategy 2: Choose Correct Formula

- CSA or TSA? (Check what's asked)
- For combinations, identify each part
- For conversions, equate volumes

Strategy 3: Unit Consistency

- Convert all to same unit (cm, m, etc.)
- Volume in cm^3 or m^3
- Surface area in cm^2 or m^2

Strategy 4: For Combinations

- Surface Area: Add visible surfaces, subtract hidden
- Volume: Simply add all volumes

Strategy 5: For Conversions

- $\text{Volume}_1 = \text{Volume}_2$
- Find unknown dimension

15. IMPORTANT POINTS TO REMEMBER

✓ For cone: Always find l first if not given

✓ Sphere has only one formula for SA ($4\pi r^2$)

✓ Hemisphere TSA = $3\pi r^2$ (not $2\pi r^2$)

✓ In conversions: $\text{Volume}_1 = \text{Volume}_2$

✓ In combinations: Subtract common areas

✓ Frustum: $r_2 > r_1$ (larger base at bottom)

✓ Cylinder: $TSA = 2\pi r(r + h)$ or $2\pi rh + 2\pi r^2$

✓ Cone volume = $(1/3) \times$ cylinder volume (same r, h)

✓ 1 litre = 1000 cm^3

✓ $1 \text{ m}^3 = 1000000 \text{ cm}^3$

16. QUICK REVISION CHECKLIST

- All formulas for 7 solids memorized
- Difference between CSA and TSA clear
- Slant height formula for cone
- Frustum formulas mastered
- Combination strategy understood
- Conversion principle clear (equal volumes)
- Water displacement problems practiced
- Cost calculation method known
- Unit conversions mastered
- All worked examples solved

ALL THE BEST! 📦📐🔴🟢🌟

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